

HETEROTIC SUPERSYMMETRY, ANOMALY CANCELLATION AND EQUATIONS OF MOTION

Stefan Ivanov

Abstract. *The bosonic fields of the ten-dimensional supergravity of the form $R^{1,9-d} \times M^d$ are the spacetime metric g , the NS three-form field strength H , the dilaton ϕ , the gauge connection A with curvature F^A . The metric connection with skew-symmetric torsion $T = H$ is $\nabla^+ = \nabla^g + \frac{1}{2}H$, ∇^g – the Levi-Civita connection of g . The heterotic equations of motion are*

$$\begin{aligned} Ric_{ij}^g - \frac{1}{4}H_{imn}H_j^{mn} + 2\nabla_i^g \nabla_j^g \phi - \frac{\alpha'}{4} \left[(F^A)_{imns} (F^A)_j^{mns} - R_{imns} R_j^{mns} \right] &= 0; \\ \nabla_i^g (e^{-2\phi} H_{jk}^i) &= 0; \quad \nabla_i^+ (e^{-2\phi} (F^A)_j^i) &= 0, \end{aligned} \quad (1)$$

A heterotic geometry preserves supersymmetry iff the following Killing-spinor equations hold:

$$\nabla_m^+ \epsilon = \nabla^+ \epsilon = 0; \quad (d\phi - \frac{1}{2}H) \cdot \epsilon = 0; \quad F^A \cdot \epsilon = 0. \quad (2)$$

The instanton equation, the last equation in (2) means that the curvature 2-form F^A is contained in the Lie algebra of a Lie group which is the stabilizer of the spinor ϵ . In dimension 5, 6, 7 and 8 the groups are $SU(2)$, $SU(3)$, G_2 and $Spin(7)$, respectively.

The Green-Schwarz anomaly cancellation mechanism requires that the three-form Bianchi identity receives an α' correction of the form

$$dH = \frac{\alpha'}{4} \left(\text{Tr}(R \wedge R) - \text{Tr}(F^A \wedge F^A) \right). \quad (3)$$

Theorem 1.1. *[1, Theorem 1.1] The heterotic Killing spinor equations and the anomaly cancellation imply the heterotic equations of motion in dimensions 5, 6, 7 and 8 iff the connection on the tangent bundle is an instanton.*

Theorem 1.1 was confirmed and reproved after in [2, 3].

For heterotic compactifications in dimension six this reduces the choice of that connection to the unique $SU(3)$ instanton on a manifold with stable tangent bundle of degree zero due to Theorem 1.1 and the celebrated result of Li-Yau about the uniqueness of the instanton.

Bibliography

- [1] S. Ivanov, Heterotic supersymmetry, anomaly cancellation and equations of motion, *Phys. Lett. B*, (2010), **685**, 190–196.
- [2] D. Martelli, J. Sparks, Non-Kähler heterotic rotations, *Adv. Theor. Math. Phys.*, (2011), **15**, 131–174.
- [3] X. de la Ossa, E. Svanes, Holomorphic Bundles and the Moduli Space of $N = 1$ Heterotic Compactifications, *J. High Energy Phys.*, (2014), No. 10, 123, DOI:10.1007/JHEP10(2014)123.

Stefan Ivanov^{1,*}

¹ University of Sofia “St. Kl. Ohridski”,
Faculty of Mathematics and Informatics,
5 James Bourchier Blvd., 1164 Sofia, Bulgaria

*Corresponding author: ivanovsp@fmi.uni-sofia.bg