ЮБИЛЕЙНА НАУЧНА СЕСИЯ – 30 години ФМИ
ПУ "Паисий Хилендарски", Пловдив, 3-4.11.2000

ON THE METRIC PROPERTIES OF SOME CONJUGATE FUNCTIONS

Ivan Hristov Feschiev

The paper is devoted to the investigation of some conjugate functions and the establishment of their main properties. As an application the optimal upper estimation of their norm in L at the most general assumptions possible for them is also proved.[1]

Introduction.

Let L (L_∞) be the space of the defined on the real axis 2π - periodic summable (essentially limited) real-valued functions with norm

(0.1) $\|f\|_L = \|f\|_1 = \int_0^{2\pi} |f(x)|\,dx$; $\|f\|_{L_\infty} = \|f\|_\infty = \sup_x vrai\ |f(x)|$.

Let $f \sim \frac{a_0}{2} + \sum_{k=1}^\infty (a_k \cos kx + b_k \sin kx)$; $\tilde{f} \sim \sum_{k=1}^\infty (-b_k \cos kx + a_k \sin kx)$ be the Fourier series expansions to $f(x)$ and to the trigonometrically conjugate to $f(x)$ function $\tilde{f}(x)$ respectively [1]. During the last decade our efforts have been intent on the solution of the following extremal problem [2]:

(0.2) $$\sup_{\|f\|_\infty \le 1} \|\tilde{f}\|_1 = 4\tilde{K}_1 ,$$

where

(0.3) $$\tilde{K}_1 = \frac{4}{\pi}\sum_{\nu=0}^\infty \frac{(-1)^\nu}{(2\nu+1)^2} = \int_0^\infty \arctan \frac{1}{\sinh(\pi y/2)}\,dy = 1.166243616...$$

is Favard's constant. This fact is closely linked with a generalization of the theorem of Stein and Weiss [3] (see also [4], th.1.10, p.27) concerning metric properties of the conjugate characteristic functions of given sets on the interval $[0,2\pi]$. This extension was made in our work [2] (th.1.2.).

The current paper appears to play the part of an addition to the investigation of the problem (0.2), (0.3) considered in [2].

Statement.

Further on we shall use the symbols and results of the articles [4, ch.I] and [2]. The letter μ is used to denote Lebesgue's measure. Since we have to work with 2π - periodic functions

[1] AMS Subject Classification: 42A50, 28A25

the short notation $T=[0,2\pi]$ turns out to be more convenient. Sets in the form of $\{x: f(x)>t,\ x\in T\}$, $\{x:\ \tau\le f(x)<t,\ \ x\in T\},...$ are to be written for brevity as follows: $[f>t]$, $[\tau\le f<t]$, …

Now let introduce the set: $\Omega=\Omega(E)=\{g\}$, where $E\subset T$ and $g(x)=\{+1 \text{ for } x\in E_1;\ 0 \text{ for } x\in E;\ -1 \text{ for } x\in E'\}$ ($E_1\subset T\setminus E$, $E'=T\setminus(E_1\cup E)$). The pivotal role in the investigation on the problem (0.2), (0.3) (see [2]) fell to the following magnitudes:

$$F\left[|\tilde g|;E\right]=\int_0^\infty \mu\left[|\tilde g|>y\right]\cap E\,dy \ ,\ (g\in\Omega) \tag{1.1}$$

$$G(\tilde g;a)=2\pi^{-1}\int_0^\infty dy\int_E arctg\{[\cosh(\pi|\tilde g(t)|/2)]/[\sinh(\pi y/2)]\}dt,\ (a=\mu E) \tag{1.2}$$

Besides for brevity we shall utilize the notation: $\mu E=a\in(0,2\pi)$.

$$A(x)=\sum_{k=1}^\infty k^{-2}\sin(kx)\ \ ;\qquad B(x)=-\sum_{k=1}^\infty(-1)^k k^{-2}\sin(kx) \tag{1.3}$$

or its equivalents

$$A(x)=-\int_0^x \ln\left[2\sin(t/2)\right]dt\ \ ;\qquad B(x)=\int_0^x \ln\left[2\cos(t/2)\right]dt\ ,\ \ (0\le x\le\pi). \tag{1.3'}$$

Lemma 1.1. For every function $g\in\Omega(E)$ the following estimate holds:

$$F\left\lfloor|\tilde g|;E\right\rfloor\le 4\pi^{-1}\{A(a/2)+B(a/2)\}\equiv\Psi_2(a)\quad(0<a<2\pi). \tag{1.4}$$

It is found [2] a sufficient condition for the solution of the problem (0.2), (0.3). This is the following inequality

$$F\left\lfloor|\tilde g|;E\right\rfloor\le G(\tilde g;a)\quad(g\in\Omega(E);\ \ 0<a<2\pi). \tag{1.5}$$

1^0. Let $g(x)=\{+1 \text{ for } x\in(0,x_1);\ -1 \text{ for } x\in(x_1,x_2);\ \ 0 \text{ for } x\in(x_2,2\pi)\}$, where $0<x_1<x_2=2\pi-a<2\pi$. The conjugate function $\tilde g$ has the representation

$$\tilde g(x)=\pi^{-1}\sum_{k=1}^\infty k^{-1}\left\{2\cos(k(x-x_1))-\cos(k(x-x_2))-\cos(kx)\right\}\ . \tag{1.6}$$

which has zeros in the points

$$\alpha_0=x_1/2-a/4+\delta\ ;\quad \beta_0=\pi+x_1/2-a/4-\delta\ , \tag{1.7}$$

$\delta=\delta(x_1;a)=\arcsin\{\sin^2(a/4)\sin^{-1}(x_1/2+a/4)\}\quad(0<x_1<2\pi-a;\ 0<a<2\pi)$.

Taking into account the fact that $\tilde g(x)$ has zero mean value on T and using the formulas (1.7) we can find the norm $\|\tilde g\|_1$:

$$\begin{aligned}\|\tilde g\|_1=2\pi^{-1}\{&2[A(\beta_0-x_1)+A(x_1-\alpha_0)]-[A(a+\beta_0)-A(a+\alpha_0)]-\\ &[A(\beta_0)-A(\alpha_0)]\}\ (0<x_1<2\pi-a;\quad 0<a<2\pi).\end{aligned} \tag{1.8}$$

On the other hand we can easily establish

(1.8) $F\left[\left|\tilde{g}\right|;E\right] = 2\pi^{-1}\{A(x_1) - A(x_1+a) + A(a)\}$ $(0 < x_1 < 2\pi - a)$.

In terms of the proven in [2] extension of the theorem of Stein and Weiss and the properties of the function $\tilde{g}(x)$ one can calculate the norm $\|\tilde{g}\|_1$ by two different means. Comparison of the obtained results leads to the equality

(1.10) $G(\tilde{g};a) = F[|\tilde{g}|;E] + 4\pi^{-1}\int_0^\infty dy \int_E arctg\{\exp[\pi(\tilde{g}(t) - y)/2)]\}dt$,

which immediately involves (1.5). In the same time the formula

(1.11)
$$G(\tilde{g};a) = 4\int_0^\infty arctg\{\sin(x_1/2 + a/4)/\sinh(\pi y/2)\}dy + F[|\tilde{g}|;E] - \|\tilde{g}\|_1$$
$$\equiv 2\Psi_2(x_1 + a/2) + F[|\tilde{g}|;E] - \|\tilde{g}\|_1$$

shows that $\|\tilde{g}\|_1 \le 4\tilde{K}_1$, $(0 < x_1 < 2\pi - a;\ 0 < a < 2\pi)$.

Remark. The stated above method for proving of (0.2), (0.3) for the considered here functions $\{g\}$ is obviously valid for every function g which conjugate $\tilde{g}$ keeps constant sign on E.

Among other things the examination shows that $\max_{x_1} F[|\tilde{g}|;E] = F[|\tilde{g}_1|;E]$, where $g_1 = g_1(a;x)$ is the function for which $x_1 = \pi - a/2$. By means similar to those applied in the investigation of the functions in examples 1-3 of [2], one can get all properties of g_1. Here we shall provide only the main formulas:

(1.12)
$$F\left[\left|\tilde{g}_1\right|;E\right] = 2\pi^{-1}\{A(\pi - a/2) - A(\pi + a/2) + A(a)\} \equiv$$
$$2\pi^{-1}\{2B(a/2) + A(a)\} \equiv 4\pi^{-1}A(a/2) \equiv \Psi_1(a) \quad (0 < a < 2\pi).$$

(1.13) $\|\tilde{g}_1\|_1 = 4\pi^{-1}\{2B(\gamma) + A(\gamma + a/2) + A(\gamma - a/2)\}$ $(0 < a < 2\pi)$

(1.14) $G(\tilde{g}_1;a) = 4\tilde{K}_1 + 4\pi^{-1}\{A(a/2) - 2B(\gamma) - A(\gamma + a/2) - A(\gamma - a/2)\}$,

where $\gamma = \gamma(a) = \arccos[-\sin^2(a/4)]$ $(0 < a < 2\pi)$.

Meanwhile the zeros of $\tilde{g}_1$ are in the points $\alpha_0 = \gamma - a/2$; $\beta_0 = 2\pi - a/2 - \gamma$.

Now we can prove the following important property

(1.15) $\min_{x_1}\{4I - \|\tilde{g}\|_1\} \equiv \min_{x_1}\{2\Psi_2(x_1 + a/2) - \|\tilde{g}\|_1\} = 4\tilde{K}_1 - \|\tilde{g}_1\|_1$.

2^0.Let $g(x) = \{+1 \text{ for } x \in (0,x_1); -1 \text{ for } x \in (x_2,x_3); 0 \text{ for } x \in (x_1,x_2) \cup (x_3,2\pi);\}$, where $0 < x_1 < x_2 < x_3 < 2\pi$ and $x_2 - x_1 + 2\pi - x_3 = a$. The conjugate function $\tilde{g}$ has the following representation

$$\tilde{g}(x) = \pi^{-1}\sum_{k=1}^\infty k^{-1}\{\cos(k(x - x_1)) + \cos(k(x - x_2)) - \cos(k(x - x_3) - \cos(kx)\}.$$

It has zeros in the points

(1.16) $\alpha_0 = \pi/2 + x_2/2 - a/4 - \nu$; $\beta_0 = \pi/2 + x_2/2 - a/4 + \nu$,

$\nu = \nu(x_1, x_2; a) = \arccos\{\sin(a/4)\sin(x_1/2 + a/4 - x_2/2)\sin^{-1}(x_1/2 + a/4)\}$,
$(0 < x_1 < x_2 < x_3 = 2\pi + x_2 - x_1 - a; \quad 0 < a < 2\pi)$.

Taking into account the fact that $\tilde{g}(x)$ has zero mean value on T by means of the formulas (1.16) we can calculate the norm $\|\tilde{g}\|_1$:

(1.17) $$\|\tilde{g}\|_1 = 2\pi^{-1}\{A(\beta_0 - x_1) + A(\beta_0 - x_2) + A(x_3 - \beta_0) - A(\beta_0) + A(x_1 - \alpha_0) + A(x_2 - \alpha_0) - A(x_3 - \alpha_0) + A(\alpha_0)\}.$$

One can prove that for a fixed $a \in (0.2\pi)$: $\max_{x_1, x_2, x_3} F\left[\|\tilde{g}|; E\right] = F\left[\|\tilde{g}_2|; E^{(2)}\right]$, where $\mu E^{(2)} = a$. g_2 is the function for which $x_1 = \pi - a/2; x_2 = \pi;$ $x_3 = 2\pi - a/2$. In this connection $F\left[\|\tilde{g}_2|; E^{(2)}\right] = \Psi_2(a) (0 < a < 2\pi)$; $\max_a F\left[\|\tilde{g}_2|; E^{(2)}\right] = 2\tilde{K}_1$ at $a = \pi$ (when $a = \pi$ the function $g_2(\pi; x)$ has the points of discontinuity: $0; \pi/2; \pi; 3\pi/2$). The norm of $g_2(a; x)$ is $\|\tilde{g}_2\|_1 = 8\pi^{-1}\{A(\pi/2 - a/4) + A(\pi/2 + a/4)\}$. It takes its largest value, equal to $4\tilde{K}_1$ at a=0.

The more important properties of the functions $\Psi_2(a)$ and $G(\tilde{g}_2; a)$ are the following [2]: $\Psi_2(x) = \Psi_2(2\pi - x) (0 < x < \pi)$; $\Psi_2'(0+0) = +\infty$, the graph of Ψ_2 is upper convex, $2\pi^{-1}\tilde{K}_1 a < \Psi_2(a)$ $(0 < a < \pi)$; $\Psi_2(a) \le 2\pi^{-1}\tilde{K}_1 a$ $(\pi \le a < 2\pi)$;

$$G(\tilde{g}_2; a) = 4\tilde{K}_1 + 4\pi^{-1}\{A(a/2) + B(a/2) - 2[A(\pi/2 - a/4) + B(\pi/2 - a/4)]\}$$

(1.18) $$\Psi_2(a) < G(\tilde{g}_2; a) \ (0 < a < 2\pi).$$

Now we can prove the following important property

(1.19) $$\min_{x_1, x_2}\{4I - \|\tilde{g}\|_1\} \equiv \min_{x_1, x_2}\{2\Psi_2(x_1 + a/2) - \|\tilde{g}\|_1\} = 4\tilde{K}_1 - \|\tilde{g}_2\|_1.$$

3^0. Let $g \in \Omega(E)$ with $\mu E = a \in (0, 2\pi)$. We construct an intermediate function $g_0^*(x)$ (taking only the values +1 and –1) for which on an appropriate subset $E_* \subset T$ holds

(1.20) $$F\left[\|\tilde{g}_0^*|; E_*\right] = 0.5\{F\left[\|\tilde{g}_0|; E\right] + F\left[\|\tilde{g}_0|; E'\right]\}, \ (E' = T \setminus E),$$

where $g_0(x) = \{+1, \ for \ x \in (0, a); -1, \ for \ x \in (a, 2\pi)\}$. Taking into account the properties of the function $\tilde{g}_0(x)$:

$$\tilde{g}_0(x) = 2\pi^{-1}\sum_{k=1}^{\infty} k^{-1}\{\cos(k(x - a)) - \cos(kx)\};$$

$$F\left[\|\tilde{g}_0|; E\right] = 4\pi^{-1}\{2A(a/2) - A(a)\}; \quad F\left[\|\tilde{g}_0|; E'\right] = 4\pi^{-1}\{A(a) + 2B(a/2)\}$$

and lemma 1.1 we establish the inequality

(1.21) $$F\left[\|\tilde{g}|; E\right] \le F\left[\|\tilde{g}_0^*|; E_*\right] \quad (g \in \Omega(E)),$$

from where follows the property: (i) either two magnitudes $F\left[\|\tilde{g}_0|; E\right]$ and $F\left[\|\tilde{g}_0|; E'\right]$ $(E' = T \setminus E)$ are both bigger or equal than $F\left[\|\tilde{g}|; E\right]$, or (ii)

$F\left[\left|\tilde{g}_0\right|;E\right] \le (or \ge)\ F\left[\left|\tilde{g}\right|;E\right] \le (or \ge)\ F\left[\left|\tilde{g}_0\right|;E'\right]$ for $\mu E \in (0,\pi)$ (or $\mu E \in [\pi, 2\pi)$) at which $F\left[\left|\tilde{g}\right|;E\right] - F\left[\left|\tilde{g}_0\right|;E'(E)\right] \le F\left[\left|\tilde{g}_0\right|;E(E')\right] - F\left[\left|\tilde{g}\right|;E\right]$. To combine two cases we turn to the function $g_0^*(x) = \{+1,\ for\ x \in (0,\xi);\quad -1,\ for\ x \in (\xi, 2\pi)\}$ in (1.20) where ξ is suitably chosen.

On the other hand with the theorem of Stein and Weiss (see th.1.1 in [2]) we can easily obtain

(1.22) $$\mu\left[\,\left|\tilde{g}_0\right| > y\,\right] \cap \begin{cases} E \\ E' \end{cases} = 4\ arctg \frac{\sin(\mu E/2)}{\exp(\pi y/2) \pm \cos(\mu E/2)}\ .$$

After integration of both sides of (1.22) from 0 to ∞ and with the help of (1.20)-(1.22) we establish the inequality

(1.23) $$F\left[\left|\tilde{g}\right|;E\right] \le 2\int_0^\infty arctg\{\sin(\mu E/2)[\sinh(\pi y/2)]^{-1}\}\,dy$$

The estimation (1.23) gives us an opportunity to make the definition

(1.24) $$F\left[\left|\tilde{g}\right|;E\right] = 2\pi^{-1}\mu E\int_0^\infty arctg\{\lambda[\sinh(\pi y/2)]^{-1}\}\,dy\,,\ (g \in \Omega(E))$$

where the constant λ is suitably chosen. The existence of λ follows from the fact that the integral in the right side of (1.24) is an increasing function of λ. Taking into account (1.24) and the property

(1.25) $$G(\tilde{g};a) \ge 2\pi^{-1}\tilde{K}_1 a\ \ (0 < a < 2\pi;\ g \in \Omega(E))$$

we make a conclusion that the inequality (1.5) will be fulfilled from every $g \in \Omega(E)$ for which $\lambda = \lambda(\tilde{g};E) \le 1$. In particular from (1.23) and (1.25) follows that (1.5) holds for every $g \in \Omega(E)$ at the condition $\mu E = a \in [\pi, 2\pi)$. Moreover subsequent upon lemma 1.1. and (1.25) immediately follows that (1.5) holds for every $g \in \Omega(E)$ satisfying $F\left[\left|\tilde{g}\right|;E\right] \le 2\pi^{-1}\tilde{K}_1 a$ (see also (1.24)) or $G(\tilde{g};a) \ge \Psi_2(a)$.

So it remains unknown only the case

(1.26) $$2\pi^{-1}\tilde{K}_1 a < F\left[\left|\tilde{g}\right|;E\right];\ G(\tilde{g};a) < \Psi_2(a)\ ,\ (0 < a < \pi)\,.$$

4^0. In this section we shall develop a new approach for proving of (1.5) when the function g has a few points of discontinuity (in general case see [2]). This method covers the above wellgrounded particular cases.

Lemma 1.2. [2] For every $g \in \Omega(E)$ and an arbitrary constant $\lambda > 0$ the following equalities hold

(1.27) $$\lambda F\left[\left|\tilde{g}\right|;E\right] = F\left[\left|\lambda\tilde{g}\right|;E\right]\ ;\quad \lambda G(\tilde{g};a) = G(\lambda\tilde{g};a)$$

where $G(\lambda\tilde{g};a) = 2\pi^{-1}\int_0^\infty dy\int_E arctg\{\cosh(\pi\left|\tilde{g}(t)\right|/2)[\sinh(\pi y/2\lambda)]^{-1}\}\,dt$.

Given a function $g \in \Omega(E)$ we define the constant $\lambda_2 > 0$ such that

(1.28) $$F\left[\left|\tilde{g}\right|;E\right] = \lambda_2\Psi_2(a) = F\left[\left|\lambda_2\tilde{g}_2\right|;E^{(2)}\right]$$

Lemma 1.1. shows that $\lambda_2 \le 1$. By means of the generalization of the theorem of Stein and Weiss (see th.1.2 in [2]) we easily reach

(1.29) $$F\left[\|\tilde{g}_2|; E^{(2)}\right] = G(\tilde{g}_2; a) + \|\tilde{g}_2\|_1 - 4\tilde{K}_1$$

(1.30) $$F\left[\|\tilde{g}|; E\right] = G(\tilde{g}; a) + \|\tilde{g}\|_1 - 4I$$

where I is the integral like those in (1.11). With the help of lemma 1.2 the equality (1.28) can be written as

(1.31) $$\{\|\tilde{g}\|_1 - \|\lambda_2 \tilde{g}_2\|_1\} - \{4I - 4\tilde{K}_1\lambda_2\} = G(\lambda_2 \tilde{g}_2; a) - G(\tilde{g}; a).$$

From (1.31) immediately follows the equivalence of the following pairs of inequalities

(1.32) $$\{\|\tilde{g}\|_1 - \|\lambda_2 \tilde{g}_2\|_1\} \le (or >) 4I - 4\tilde{K}_1\lambda_2 \to (or \leftarrow) = G(\lambda_2 \tilde{g}_2; a) \le (or >) G(\tilde{g}; a).$$

By means of the property (1.18) and lemma 1.2 we easily get

(1.33) $$F\left[\|\tilde{g}|; E\right] = \lambda_2 \Psi_2(a) < \lambda_2 G(\tilde{g}_2; a) = G(\lambda_2 \tilde{g}_2; a)$$

Now we consider the two pairs of inequalities

(1.34) a) $G(\lambda_2 \tilde{g}_2; a) \le G(\tilde{g}; a)$; b) $F\left[\|\tilde{g}|; E\right] < G(\tilde{g}; a)$

(1.35) c) $G(\lambda_2 \tilde{g}_2; a) > G(\tilde{g}; a)$; d) $F\left[\|\tilde{g}|; E\right] \ge G(\tilde{g}; a)$.

It occurs obviously that a) $\to$ b) and d) $\to$ c). Moreover the systems of inequalities (a,d) and (b,c) are separately incompatible. It is apparently true for the first system. For the second one its incompatibility grounds on the fact that on the set of all functions $g \in \Omega(E)$ satisfying the condition b), the difference $4I - \|\tilde{g}\|_1$ takes its smallest value (equal to $4\tilde{K}_1 - \|\tilde{g}_2\|_1$) at $g = g_2$. The proof of this is based on the properties: (1.15),(1.19), some ideas in the proof of lemma 1.1 [2] and the inequality $G(\tilde{g}_2; a) - \Psi_2(a) < G(\tilde{g}_1; a) - \Psi_1(a)$ $(0 < a < 2\pi)$. So the pairs of inequalities both in (1.34) and in (1.35) are equivalent.

Various examples show that for every fixed $a \in (0, 2\pi)$ there exists an uncountable set of functions (particularly satisfying (1.26)) for which the correlations (1.34) hold. This fact is significant for the following assertion: if for a fixed a we assume the existence of a function $g \in \Omega(E)$ for which holds (1.35) then by deformation of g (by means described in the proof of lemma 1.1. [2]) we can ensure both an uncountable set of functions g satisfying (1.35) and an uncountable set of functions g satisfying (1.34). Keeping the above assumption let we choose an infinite sequence $\{\tilde{g}_n\}$, convergent to $\tilde{g}_0$, such that

(1.36) $$G(\lambda_2^n \tilde{g}_2; a) > G(\tilde{g}_n; a) \ (n = 1, 2, \ldots); \ G(\lambda_2^0 \tilde{g}_2; a) = G(\tilde{g}_0; a)$$

(At the assumption $g_0 = -(\tilde{g}_0)^{\sim} \,\overline{\in}\, \Omega$ one can reach a discrepancy). Subsequent upon the equivalence (1.35) we will have $F\left[\|\tilde{g}_n|; E^{(n)}\right] \ge G(\tilde{g}_n; a)$ $(\mu E^{(n)} = a)$, from where after a limit passage we will obtain $F\left[\|\tilde{g}_0|; E^{(0)}\right] \ge G(\tilde{g}_0; a)$ $(\mu E^{(0)} = a)$.

On the other hand the equivalence (1.34) immediately leads to the inverse inequality $F\left[|\tilde{g}_0|; E^{(0)}\right] < G(\tilde{g}_0; a)$. Also we can reach a discrepancy if we take the strong inequality b). So there is no boundary case. In other words there is no function $g \in \Omega(E)$ satisfying $G(\lambda_2 \tilde{g}_2; a) = G(\tilde{g}; a)$ or $F\left[|\tilde{g}|; E\right] = G(\tilde{g}\ ; a)$. Every function $g \in \Omega(E)$ can satisfy only (1.34) or (1.35) without equality. Practical examples show that only (1.34) is valid.

Conclusion.

In this paper there is proved the following [2]

Theorem. If $f \in L_\infty$ and $\|f\|_\infty \le 1$ the following estimation holds

(1.37) $\|\tilde{f}\|_1 \le 4\tilde{K}_1 = 4.664974464...$

The equality is achieved for all functions $f \in \Omega(E)$ $(\mu E = 0)$ which satisfy the condition $\mu[f > 0] \cap T = \pi$.

REFERENCES

1. A.Zygmund. Trigonometric Series. Vol.I. Cambridge at the University Press, (1959).
2. I.H.Feschiev and S.G.Gocheva-Ilieva. On the extension of a theorem of Stein and Weiss and its application (submitted for publication on May 27, 1999).
3. E.M.Stein and G.Weiss. An extension of a theorem of Marcinkiewicz and some of its applications. *J.Math. and Mech*., vol.8, no 2,(1959), pp.263-284.
4. B.V.Hvedelidze. The method of Cauchy-type integrals in the discontinuous boundary-value problems of the theory of holomorphic functions of a complex variable. *J.Sov.Math.* vol.7, (1977), pp.309-414 (translation from *Itogi Nauki Tech.*, Ser. Sovrem. Probl. Math., vol.7, (1975), pp. 6-162).

Plovdiv University "Paisij Hilendarski",
Department of Mathematics and Informatics,
24, Tzar Assen St.,
4000 PLOVDIV, BULGARIA