DEVELOPING GEOMETRIC THINKING SKILLS THROUGH DYNAMIC DIAGRAM TRANSFORMATIONS

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ABSTRACT

The paper draws on an experiment conducted in a secondary school mathematics classroom in Greece which sought to investigate the transformation of students' skills following the use of the Sketchpad dynamic diagram transformations by comparing pre and post paper-pencil tests. We conclude that the dynamic geometry software is an effective component in the conceptualisation of the solution to the problem through reasoning and can help students develop higher-order thinking skills.

DYNAMIC TRANSFORMATIONS

The paper draws on an experiment conducted in a secondary school mathematics classroom in Greece aiming to investigate the difficulties a pair of students encountered when trying to generate diagrams in a problem-solving situation, and how a DGS environment could be "an important and effective component in the conceptualisation of the problem structure, which is a critical step towards a successful solution" (van Essen & Hamaker, 1990). The mathematical problem the students engaged with, either in a DGS or static environment, generated potentially insightful data on the use of dynamic transformations focused on, in the comparison between the pre and post paper-pencil tests, regarding the students' skills improvement. De Villier (1993) supports that "students' use of transformations can deduce properties of figures". According to Coxford and Usiskin (1975), transformations are used in the teaching and learning of mathematics because they "simplify the mathematical development" and "can be understood by students of widely varying abilities". Recent research has shown that students' ability to transform geometric objects is related to their efficiency in numeracy, in particular, addition/subtraction strategies (Wheatley, 1998). Geometer's Sketchpad (Jackiw, 1991) or Cabri II (Laborde et al., 1988) software packages are highly effective visual dynamic tools for exploring and discovering the geometrical properties of Euclidean geometry objects on diagrams produced on the computer screen. Straesser (2001) claims that "…DGS-use widens the range of possible activities, provides an access route to deeper reflection and more refined exploration and heuristics than in paper and pencil geometry." Hollebrands (2003) declares that "students learning geometric transformations in a technological context may develop understandings that are influenced by their interactions with the technological tools". Many researchers who used the Sketchpad software have conducted studies on transformations using the van Hiele model as a descriptor for their analysis and have concluded that students achieved significantly higher scores between the pre- and post-tests or significantly outperformed their peers who had received traditional instruction (see for example Dixon, 1996). The van Hiele model, distinguishes five different levels of thought which are: Recognition (Level 1), Analysis (Level 2), Informal deduction (Level 3), Formal deduction (Level 4) and Rigor (Level 5) (Fuys et al., 1988). In addition, geometric thinking is inherent in the types of skills proposed by Hoffer (1981) briefly reported by Dindyal (2007) as follows: "(1) visual skills - recognition, observation of properties, interpreting maps, imaging, recognition from different angles, etc.; (2) verbal skills - correct use of terminology and accurate communication in describing spatial concepts and relationships; (3) drawing skills-communicating through drawing, ability to represent geometric shapes in 2-d and 3-d, to make scale diagrams, sketch isometric figures, etc.; (4) logical skills - classification, recognition of essential properties as criteria, discerning patterns, formulating and testing hypothesis, making inferences, using counter-examples; and (5) applied skills - real-life applications using geometric results learnt and real uses of geometry, as for designing packages, etc."

DIAGRAMS AND DYNAMIC GEOMETRY

Diezmann (2005) states that diagrams have three key cognitive advantages in problem solving: "First, diagrams facilitate the conceptualisation of the problem structure, which is a critical step towards a successful solution (van Essen & Hamaker, 1990). Second, diagrams are an inference-making knowledge representation system (Lindsay, 1995) that has the capacity for knowledge generation (Karmiloff-Smith, 1990). Third, diagrams support visual reasoning, which is complementary to, but differs from, linguistic reasoning (Barwise & Etchemendy, 1991)."

Experience in class has shown that a student with level one (or two) on the van Hiele model "often fails in the construction of a geometric configuration which is essential for the solution of the underlying geometric problem" (Schumann and Green, 1994). Moreover, students' diagrams "provide an insight into the strengths and weaknesses of their mathematical knowledge" (Diezmann, 2000). Laborde (2005) claims that dynamic geometry software supports "a new kind of diagram, because (the diagrams) result from sequences of primitives expressed in geometrical terms chosen by the user …and (are) modified according to the geometry of their constructions rather than the wishes of the user". During the construction of (or action on) a dynamic diagram the student structures an internal invisible side of the representation which is a part of the process on the external representation/model. According to Jackiw and Finzer (1993) "Students using GSP acquire an informal understanding of some of these terms…. More importantly, they come to understand how a geometric construction can be defined by a system of dependencies." This is in accordance with what Noss and Hoyles (1996) support that "in a computing environment 'students' activity is shaped by the tools' (in our case the dynamic diagrams), 'while at the same time they shape' the dynamic diagrams 'to express their arguments". Transformations which occur due to Sketchpad techniques have a significant impact: during the instrumental approach, the student structures utilization schemes (Rabardel, 1995) of the tools, and consequently mental images of the transformational processes, since any modification of the initial figure (input) results in the modification of the final figure (output).

RESEARCH METHODOLOGY

The didactic experiment was conducted in a class at a public high school in Athens during the second term of the academic year and involved 28 students aged 15-16. Firstly, the researchers examined student's level of geometric thought using the test developed by Usiskin (1982) at the University of Chicago which is in accordance to the van Hiele model. The methodology of the class experiment discussed in this paper includes an exploration of an open problem by a pair of students in the experimental group. It is divided into three parts: the first deals with the diagrams produced by the students in a paper–pencil problem pre-test solving process, the second with the exploration of the problem using dynamic geometry software, and the third with the post-test and a description of the way in which students generated diagrams and reasoned in the problem after it was reformulated by the researcher. The discussions were videotaped and examined simultaneously with the interviewers' field notes during the inquiry process. The analysis of the results that follows is based on observations in class and of the video. The participants are M_5 (van Hiele level: 2) and M_6 (van Hiele level: 2 transition to 3), both male students. The van Hiele levels of the students are used as a descriptor for the analysis. Within this theoretical framework, the following research question is posed: Do the transformational processes employed in the software during collaborative problem-solving lead to the transformation of students abilities (as described by Hoffer)?

Part 1:

The problem situation explored by the students was the revised version of the “lost treasure of the pirates” problem conceived by the Russian, George Gamow (1948, reprinted 1988). Gamow proposes a problem suggested by a treasure map found in a grandfather’s attic: “You walk directly from the flag (point F) to the palm tree (Point P), counting your paces as you walk. Then turn a quarter of a circle to the right and walk the same number of paces. When you reach the end, put a stick in the ground (point K). Return to the flag and walk directly to the oak tree (point O), again counting your paces and turning a quarter of a circle to the left and going the same number of paces. Put another stick in the ground (point L). The treasure is buried at the midpoint between the two sticks (point T) (Figure 1). After some years, the flag was destroyed and the treasure could not be found through the location of the flag

Can you find the treasure now or is it impossible?” (cf. Scherr, 2003; Patsiomitou & Koleza, 2008; Patsiomitou, 2008).

Field note 1:

Diagrams 1, 2 were generated by the students during the experimentation and problem solving process in the paper-pencil pre-test. They tried, but were unable to make any headway towards finding a solution. The researcher then helped them by constructing the outlines of the problem in Figure 1 on the blackboard. In figures 2, 3, we can see that the students constructed a number of intermediary lines that could well serve as an obstacle to their solving the problem, while they did not transform the segments correctly and transferred their mental approach to the drawing incorrectly.

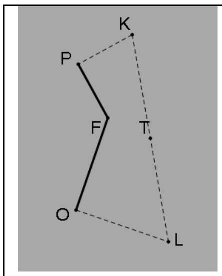	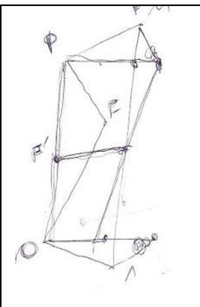	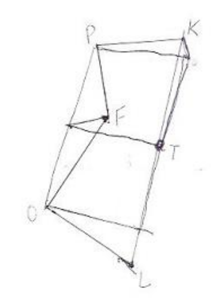
Fig.1	Fig.2	Fig.3

The diagrams they produced were incorrect. The rotation was incorrectly constructed, as the two resulting segments were not equal. For example: Student M_5 joined points P, O in Figure 2. The rest of his solution and proof lacked any underlying rationale: he mentioned not a rectangle, but a shape (it is not clear what shape exactly). Student M_6 did not understand the rotation of the segment through 90º, since the segments rotated around the midpoint were not equal (Figure 3). He was unable to continue because he had drawn a parallel line to PO which coincided with KL.

Part2:

Field note 2:

M_5 tried to start by using the mouse. M_5 connected points P and O, and then constructed two lines perpendicular to the endpoints of segment PO (Fig. 4). He tried to construct two almost equal segments, like diagonals in a parallelogram, then stopped. This was a crucial point for pupil M_5, who probably believed that point T was at the intersection of the diagonals of any parallelogram. The student's misinterpretation of the theory led to an obstacle. At this stage, cognitive conflict stemmed from a misconception of what is known from theory contrasting with what was shown on the screen. Having failed, he connected points P and O with T and constructed a perpendicular line which passed through point T on segment PO. Observing PO, the students determined that the segments PT and TO were of equal length (Fig. 5). M_6 then measured the segments PG and GO, determining that they were equal. This latter action was necessary, because it helped them understand that GT was perpendicular bisector and point G would be the midpoint of the segment PO. They decided to start again. M_6 decided to make two parallels that cut randomly through the plane at points P and O. M_5 seemed to be waiting his turn.

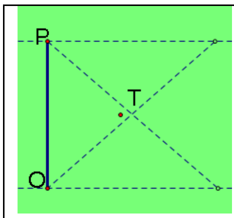

Fig.4

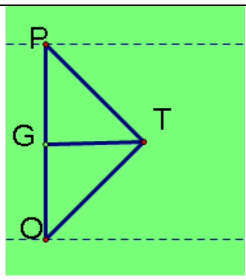

Fig.5

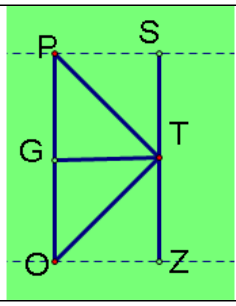

Fig.6

M_5: *Why don't we try to begin* ***inversely****?* ***If*** *we start from midpoint of PO and rotate the two segments PG, GO… So* ***if*** *we join the points S, Z* **then** *the segment will pass through the point T.* (Fig. 6)

M_5, M_6: *All the triangles that have been shaped are special rights* (isosceles and right)

M_5: *The angles are all equal to 45^o and the quadrilateral PGTS is a square.* ***Then point T is in the middle of the distance of SZ*** *which is equal with PO.*

Field note 3:

Through the experimental process the students were able to build up instrumentation schemes that combined technical and conceptual aspects. An observation of their answers would indicate that the software helped the students answer at a “higher level” than that indicated by the van Hiele test. For example, students M_5 would seem to belong to level 4, since he was “starting to develop longer sequences of statements and beginning to understand the significance of deduction” (de Villiers, 2004). Student M_5 formulated a conjecture, reversing the stream of thought. He constructed an “if...then” statement through inferences made, and student M_6 reached logical conclusions on the problem by correlating the theorems they already knew. The dynamic transformations conducted on diagrams in the software helped the pupils overcome their obstacles and prove the specific instance of the problem.

Part 3:

In a session where the problem had been discussed during a classroom learning process, the researcher assumed that insufficient clarification had been given to the students in the initial paper and pencil test. The students did not return to the problem for a whole month. Then the researcher decided to readdress the same problem. She re-formulated the problem using a different way of approaching it which experimental group students had come up with during the sessions. The reformulated problem was (Patsiomitou, 2008): “An archaeologist has an old map which explains the position of a vessel: You walk directly from point P to point F (F, E are constant points) counting your paces as you go. Then turn right 90 degrees and walk the same number of paces from point F. When you reach the end, put a stick in the ground. Return to point P and walk directly to point K, again counting your paces and turning left 90 degrees and walking the same number of paces. Put another stick in the ground. The vessel is buried in the middle of the distance of the two sticks. Rejecting the procedure described above, the archaeologist did the following: starting from the midpoint of the segment FE, he followed the directions given on the map until he finally found the pot. a) Can you plot the shape according to the steps that archaeologist followed? And b) can you explain/reason why he was right?”

Field note 4:

We shall limit ourselves to analysing the behaviour of the two students. We shall therefore examine and analyse the mental approach taken by the two students to proving the solution to the problem. Student M_5 had not produced correct constructions in earlier tests. His diagram (Fig. 7) revealed that he successfully 1) generated the correct diagram and 2) proved the congruency of the triangles to justify his solving process.

This comparison revealed that the deliberate and conscious use of the rotation command gave observable parts of the instrumentation schemes. He proved the problem for the specific case of the rectangle by recognizing the congruent isosceles and right triangles, highlighting the angle of 45 degrees on the diagram. Student M_6’ drawing was complex. Connecting P and P′, he compared the two triangles by using the criteria for triangles' congruence and performed

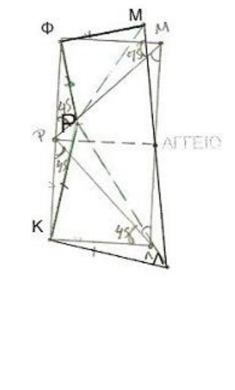	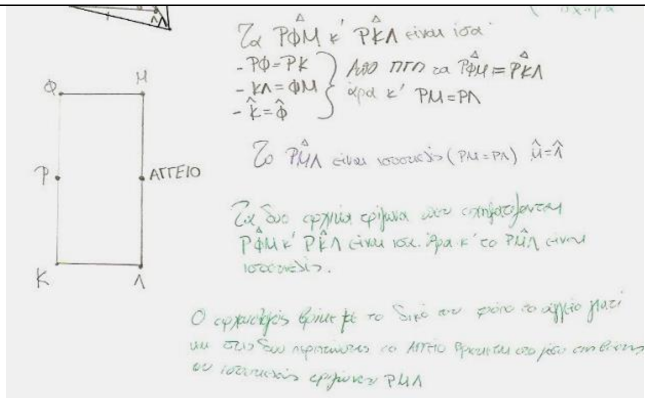
Fig.7	Fig.8
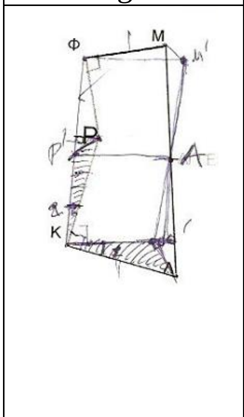	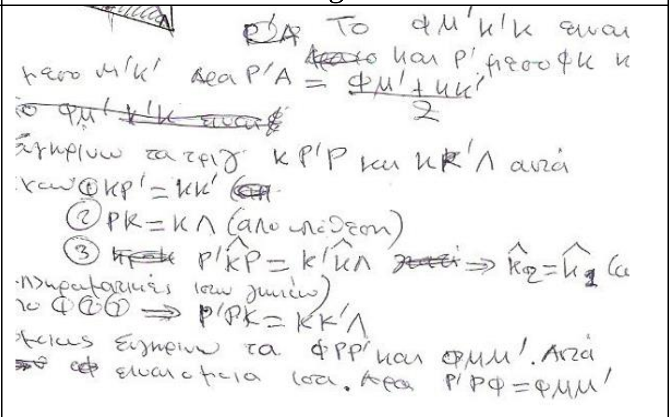
Fig.9	Fig.10

competently and adequately the acts required by the situation. He made a comparison of the two triangles by using the criteria for triangles' congruence and performed competently and adequately the acts required by the situation. His comparison was accurate and confirmed that the student had constructed a mental image of the way in which the triangle rotated, and this had helped him to solve the problem by using generalization.

DISCUSSION

The post-test results indicated that students were seemingly able to transfer the skills and the different way of looking at the reformulated problem acquired when working in Sketchpad. The students had developed their visual and verbal skills, meaning their ability to observe the properties of shapes, to recognize equalities of angles and to interpret the diagram into a verbal mapping, using terminology correctly and describing the relationships between figures. They had also developed their drawing skills and their ability to interpret the problem statement into an accurate and correct diagram; as a result, students were able to correlate the theorems they knew and tried to prove the problem using essential criteria like the congruence of triangles, formulating hypothesis and making inferences. This led us to conclude that students had improved their logical skills. Moreover, through the diagram and using the relevant theorems, the students had been guided to make logical conclusions relating to the relationships linking the geometric objects. Their latter answers showed that the dynamic diagram had made it easier for them to conceptualize the problem structure by operating as an inference-making knowledge representation system - a generator of knowledge according to Diezmann's key cognitive advantages. We agree with Schumann and Green that students in the first part faced failure in the construction of the geometric configuration they needed if they were to solve the problem. Dynamic diagrams and the students' exploration of the problem using dynamic means had helped them overcome a misconception and allowed them to interpret the problem. This means that the students had developed thinking processes and applied skills, developing a mathematical model to interpret the realistic problem. Finally, we observed a link between visual and formal abilities which was essential for the transition from the lower levels, to the upper ones.

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