ON USING A DYNAMIC GEOMETRY SOFTWARE IN THE COMPREHENSION OF THE SIMILARITY OF TRIANGLES

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ABSTRACT

In the present work we make a comparison between teaching the similarity of triangles with the Geometer's Sketchpad software and teaching the same subject in the classical way. The research took place with 3rd Year Junior Secondary School pupils. It was followed by an evaluation of the level of comprehension of the elements they had been taught, and an improvement in the pupils' comprehension was seen after the use of Geometer's Sketchpad.

INTRODUCTION

Technology can provide mechanisms to sustain assistance to mathematics teachers in their use of technology to implement mathematics education reforms in their classes. Technology enables mathematics education reform, but it is not reform per se (cf. Kaput, 1992).

Features of technology, whether mathematics-specific or more generic, should be introduced and illustrated in the context of meaningful content-based activities. Teaching a set of technology or software-based skills and then trying to find mathematical topics for which they might be useful is comparable to teaching a set of procedural mathematical skills and then giving a collection of "word problems" to solve using the procedures. Such an approach can obscure the purpose of learning and using technology, make mathematics appear as an afterthought, and lead to contrived activities. The use of technology in mathematics teaching is not for the purpose of teaching about technology, but for the purpose of enhancing mathematics teaching and learning with technology (Garofalo, Drier, Harper, and Timmerman 2000) In the early phase of our work, we devised a set of guidelines to shape our development of mathematics activities and materials (Garofalo, Shockey, Harper, & Drier, 1999). The five guidelines below reflect what we believe to be appropriate uses of of technology in mathematics teaching:

- introduce technology in context
- address worthwhile mathematics with appropriate pedagogy
- take advantage of technology
- Connect mathematics topics
- incorporate multiple representations

We very often meet the concept of resemblance in our daily life. We use it in practical problems, such as for the measurement of the distance of inaccessible points or in the manufacture of similar forms (prospect manufacture) that is used by the designers and painters in order to portray an object of three-dimensional space on a two-dimensional surface. The concept of similarity constitutes part of the science of Mathematics and particularly of the Geometry and it is taught according to the school analytic program in the High school and in the Lyceum. Its comprehension consequently has important interest. The students face difficulties that are durable and are even presented in the graduates of Lyceum (Tsiakana,2006). A lot of problems of comprehending the concept of similar triangles forms exist and particularly of the triangles in all the degrees of education. These difficulties are due to the fact that the concept of similarity is complex as it is closely connected with the significances of proportion and reason. In this project a planning of educational activities for the teaching of similar triangles with educational software of Dynamic Geometry (Sketchpad) was done to students of the third grade of High school. The objective was the students to be given the opportunity to study and comprehend the concept of resemblance with an energetic qualitative way, to be involved in a scientific research that strengthens their critical and creative faculty as well as the undertaking of initiatives. (Tsiakana,2006) Each of these guidelines is discussed below and illustrated with one or more of our activities Regarding the similarity of triangles, two triangles are similar if they have two equal angles, one to one, or have two proportional sides and their angles equal. Finally, they are similar if their sides are proportional.

Research questions: If the use of Geometer's Sketchpad increases pupils' comprehension of the concept of the similarity of triangles.

THEORETICAL FRAMEWORK

The theoretical framework that guides our work is based on a constructivist perspective in which learning is viewed as a process of experiencing dissonance and working to resolve perturbations by building viable explanations (von

Glasersfeld, 1987, 1995 Bowers J. and Doerr H 2001). As Steffe and Thompson (2000) recently noted, Piaget contended that there are four factors that contribute to one's cognitive development. These include social interaction, maturation, physical experience, and self-regulation. "Individuals establish equilibrium among personal schemes of action and anticipation as they interact in mutual adaptation – as constrained by local limitations imposed by their abilities to accommodate those very schemes" (Steffe & Thompson, 2000, p. 193 Bowers J and Doerr H (2001)). The critical element of this general model is that students are seen as cognizing individuals who are continually interacting with each other and with their environment (which includes the computer and the accompanying activities) and adapting their own views through processes of interactive accommodation. We take very seriously Steffe and Thompson's recommendation that "Researchers should not apply general models like von Glasersfeld's or Vygotsky's directly to the practice of mathematics education" (p. 204). In fact, we view the model as a general way of looking at how the participants in our study accommodated their current ways of knowing mathematics with the unanticipated outcomes they experienced during some of their activities. To create the need for our participants to adapt their mathematical understandings and their views of effective pedagogy, we designed activities that were grounded in the quate when teaching mathematics in ways that support more meaningful understanding (Bowers and Doerr, 2001).

A well-designed software programme, under the direction of the teacher, allows students to test and investigate concepts so that their comprehension develops as a continuous and dynamic process through observation, reflection and experimentation. The objects that are produced on the screen can be symbols for common reference and discussion between the teacher and the students (National Council of Teachers of Mathematics, 2000).

Moreover, the computer in mathematics can encourage, hypothesis, justification and generalization, and allow rapid and precise calculation, analysis of elements, reasoning, and investigations of numerous representative types (e.g. numeric, symbolic, and graphic). According to the socio-cultural picture, technology is considered as being one of various types of cultural tools which not only strengthen but also reorganize the cognitive processes, through their integration into social practice (Tamis, 2005).

Before a teacher begins his lesson he is face to face with his students and must take them into account as elements which he must use to adapt his teaching. Other elements which he must take into account are the teaching classroom, any problems of communication which may then be in the classroom or area where he is teaching, the supervisory means of his teaching, and the computers which the students and teachers have at their disposal. Apart from this there are elements which have to do with the students themselves which the teacher can use to help them. This will be to do with the way in which they can face up to their fears and negative thoughts about technology. It can also lead them to learn by themselves (Hall et al., 1977).

METHODS

34 students participated in the research, which was qualitative. The students were divided into two groups, the experimental group and the control group. Each group contained 17 students. The experimental group was taught using the Geometer's Sketchpad software while the control group and experimental group was taught in the classical way of teaching. The groups had to be of equal ability in terms of knowledge. For this reason the geometry teacher's evaluation was taken into account. It should be noted that neither group had previously had teaching on the similarity of triangles.

Then the teacher called upon the control group to proceed as follows:

Researcher: Create a triangle called BCD.

Students': How?

Researcher: Select the triable: To do this, hold down the shift key, then click on each vertex and each side. From the Transform menu, choose Dilate. When the dialogue box appears on the screen, type 2.00 in the New box and 1.00 in the Old box, if these numbers do not already appear. Click Ok. You should now see B′C′D′ (figure 1)

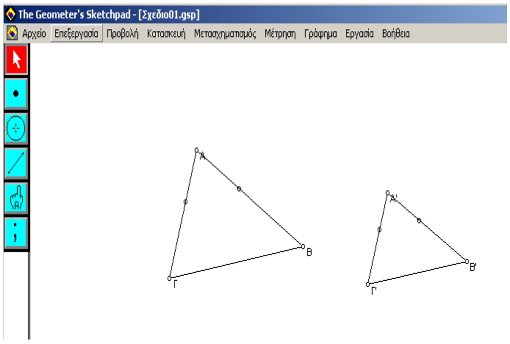

Figure 1

Students' action: The students followed the teacher's instructions and created a triangle.

Researcher: What do you have to say about the two triangles?

Students :These two triangles are similar triangles and we show this by writing them as:BCD -B′C′D′

Researcher:Very good

Researcher: We will see that when the corresponding angles in similar triangles are equal:

Students: It will be necessary to measure the angles

Researcher: Ok

Students : It' s same

Researcher:

a. Measure the angles CDB and C′D′B′ and DBC and D′B′C′

b. Drag the vertices of the two triangles to change their shapes and sizes. What do you notice about the angle measures? Explain

Students' action: The students could very easily prove that when we have corresponding angles in similar triangles the triangles are the same.

B. Corresponding Sides in similar triangles.

Researcher: Measure segment BC using and Measure segment B′C′.How does the length of B′C′ compare to the length of BC?

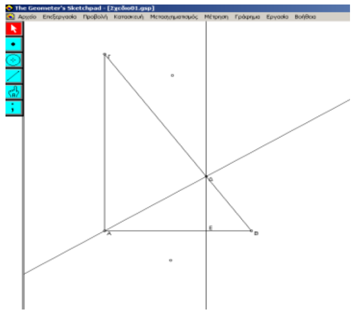

Figure 2

Students: To measure the ratio of the lengths directly, select sides B′C′ and BC at the same time and then from the Measurement menu, choose Ratio.
Researcher: Repeat last step to compare the measures of the sides C′D′ and CD, then sides D′B′ and DB.
Students' action: The students make the comparison that the researcher asked for.
Researcher: What do you notice about the ratios;

Students $\frac{B'C'}{BC}, \frac{C'D'}{CD}$ and $\frac{D'B'}{DB}$

Researcher: Drag the vertices of the two triangles to change their shapes and sizes. What do you notice about the ratios of corresponding sides? Explain.

Next we must calculate if there is proportionality between the surfaces of the two triangles.
Researcher: To calculate the area of BCD, select points B, C, and D at the same time. From the Construct menu, choose Polygon Interior. From the Measure menu, choose Area
Calculate the are of B'C'D'.

To calculate the ratio $\frac{area\ of\ B'C'D'}{area\ of\ BCD}$, click on the interior of B′C′D′.

From the Measure menu, choose Calculate.
Click on the measurement of the are of B′C′D′. In the calculator display window, (Area B′C′D') should appear. On the calculator buttons, click on the divide sign. Click on measurement of the area of BCD, then click OK.
Then the following exercise was set for both groups:
Consider the right-angled triangle ABC (A=90°), the height of AD and DE⊥AB. Show that $AD^2=AC\cdot DE$.
When the students create the picture which appears in the neighbouring diagram they can very easily show that
The angle E=D=90°
and $D_1=A_1$ hence DE//AC

hence $\frac{AD}{DE}=\frac{AC}{AD}$ or $AD^2=AC\cdot DE$.

Evaluation.

After 3 hours the two groups were evaluated in questions similar to the above, while at the same time they were set the question: if two triangles ABC and A'B'C' are similar and the sides a and a' are proportional, show that

$$\frac{\mu_\alpha}{\mu_\alpha} = \frac{\upsilon_\alpha}{\vartheta_\alpha}$$

The results of evaluation are distinguished below:

	They recognise the similarity by the proportional sides	They recognise the similarity when two triangles have to angles the same	Two proportional sides and containing equal angles are similar triangles	They correctly solved the exercise
Control Group	14	4	3	1
Experimental Group (taught with Geometer's Sketchpad)	17	14	12	7

Table 1

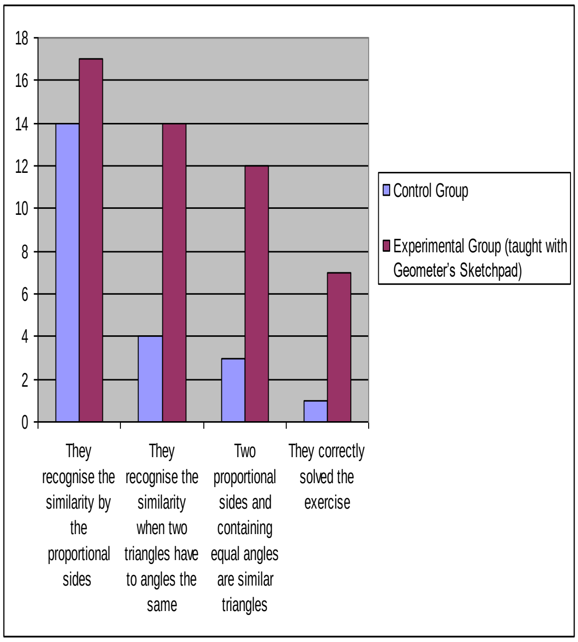

Figure 3

The correlation factor which exists between teaching in the classical way and teaching with the new Technologies is Correlation: 0.84

CONCLUSION

We must provide mathematics teachers extended opportunities to experience and do mathematics in an environment supported by diverse technologies (Dreyfus & Eisenberg, 1990). The heart of our approach is the development of mathematical power-- understanding, using, and appreciating mathematics. Our interest is in empowering teachers through the use of technology in mathematics exploration, open-ended problem solving, interpreting mathematics, developing understanding, and communicating about mathematics (Bransford, et al,1996, 1992)
We, as mathematics educators, should make the best use of multiple representations, especially those enhanced by the use of technology, encourage and help our students to apply multiple approaches to mathematical problem solving and engage them in creative thinking" . The most important aspect of "The Geometer's Sketchpad" is its ability to deal with the immediate management of mathematical objects and shapes and to edit geometrical concepts in their entirety and from different optical angles. The paedagogical approach to the software supports the opinion that learning presupposes the active participation of the students in the social framework of the class that fundamentally consists of the teacher (through the instructive intervention), the student, and the interaction of students with the instructive tools (e.g.the computing environment etc). This presupposes that the teaching environment is rich and has opportunities for the acquiring of skills, for investigation, analysis and composition, comprehension of concepts through the organization of intellectual structures, and the active interaction of the student offered tools and teaching.

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