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FLUXONS CREATION IN JOSEPHSON JUNCTIONS

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Abstract. The static distributions in a system of two multiply connected Joshepson junctions with equal lengths and different amplitudes of Josephson currents 0-j_cJJ is modeled analytically as well as numerically. A particular case of the studied systems is the 0-πJJ junction. The exact analytic solutions are constructed on an "infinite" 0-j_cJJ by the help with the oneflaxons solutions in homogeneous and j_c junctions.

The existence of this structure of $\mathcal{C}^1$-smooth distributions (semiflaxons) is demonstrated. The distributions are considered as a result of a nonlinear relation of flaxons in sub-domains of their common bound.

Key words: Josephson junction, static distribution of magnetic flux, nonlinear boundary value problem, semiflaxon, Cauchi problem with an additional candition

Mathematics Subject Classification 2000: 74K30

1. Statement of the problem

The static distributions ([1]) of magnetic flux $\varphi(x)$ in one layer junction with different amplitudes of Josephson currents $j_c(x)$ satisfy the following nonlinear boundary value problem

$$-\varphi_{xx} + j_c(x)\sin\varphi - \gamma = 0, \quad x \in (-L, L)\,, \tag{1.1a}$$

$$\varphi_x(\pm L) = h_e\,, \tag{1.1b}$$

where h_e is the outer magnetic field, γ is the outer current.

The solutions of (1.1) depends on the physics' coordinate x, as well as on the parameters L, h_e, γ and j_c, i.e., $\varphi = \varphi(x, p)$, where by $p \equiv \{L, h_e, \gamma, j_c\}$, $p \in \mathcal{P} \subset \mathbb{R}^3$ is denoted the 4-vector of the parameters of the model.

Note that in the paper, we will write the dependence on p only in the case when it is necessary.

The function $j_c(x)$ models possible non-homogeneous type of barrier layers. For a homogeneous junction the equality $j_c(x) \equiv 1$ holds. In the case of junctions of non homogeneous type with a length 2δ at the center of the junction the following relations are true:

$$j_c(x) = \begin{cases} 1, & x \notin (-\delta, \delta), \\ j_c, & x \in (-\delta, \delta). \end{cases} \tag{1.2}$$

At points $x = \pm\delta$ the solutions of (1.1) are C^1-smooth .

For multiply connected junctions we could write

$$j_c(x) = \begin{cases} 1, & x \notin (-l, 0), \\ j_c, & x \in (0, l). \end{cases} \tag{1.3}$$

where the parameter $j_c \in [-2, 2)$ and at point $x = 0$ the solutions of (1.1) are C^1 smooth. For $j_c = -1$ the problem reduces to a problem for 0-πJJ -junction.

The static distributions of the magnetic flux in Josephson systems of two multiply connected Josephson junctions with equal lengths and different amplitudes of Josephson currents 0-j_cJJ is described by the following nonlinear problem:

$$-\varphi_{0,xx} + \sin\varphi_0 - \gamma = 0, \quad x \in (-l, 0), \tag{1.4a}$$

$$-\varphi_{j_c,xx} + j_c \sin\varphi_{j_c} - \gamma = 0, \quad x \in (0, l). \tag{1.4b}$$

The magnetic flux $(\varphi_0, \varphi_{j_c})$ and the inner magnetic field $(\varphi_{0,x}, \varphi_{j_c,x})$ at the center $x = 0$ satisfy the following conditions for continuity

$$\varphi_0(0) - \varphi_{j_c}(0) = 0, \tag{1.5a}$$

$$\varphi_{0,x}(0) - \varphi_{j_c,x}(0) = 0. \tag{1.5b}$$

For a structure with a finite length $l < \infty$ in the case of geometry with overlapping ([1]) the boundary conditions

$$\varphi_{0,x}(-l) = h_e, \tag{1.6a}$$

$$\varphi_{j_c,x}(l) = h_e, \tag{1.6b}$$

hold.

The relations (1.4) - (1.6) define a nonlinear boundary value problem, corresponding to the considered model of 0-j_cJJ .

The stability of any distribution of the magnetic flux $(\varphi_0(x,p), \varphi_{j_c}(x,p))$ in 0-j_cJJ for various parameters p ([3]), are defined by the sign of the minimal eigenvalue λ_{min} of the corresponding Sturm-Liouville problem

$$\psi_{0,x}(-l) = 0, \tag{1.7a}$$

$$-\psi_{0,xx} + \cos\varphi_0\,\psi = \lambda\psi_0, \quad x \in (-l,\zeta), \tag{1.7b}$$

$$\psi_0(0) - \psi_{j_c}(0) = 0, \tag{1.7c}$$

$$\psi_{0,x}(0) - \psi_{j_c,x}(0) = 0, \tag{1.7d}$$

$$-\psi_{j_c,xx} + j_c\cos\varphi_{j_c}\,\psi = \lambda\psi_{j_c}, \quad x \in (\zeta,l), \tag{1.7e}$$

$$\psi_{j_c,x}(l) = 0 \tag{1.7f}$$

$$\int_{-l}^{0} \varphi_1(x)dx + \int_{0}^{l} \varphi_2(x)dx - 1 = 0.. \tag{1.7g}$$

On the finite interval $[-l,l]$ the problem (1.7) has bounded from below discrete specter $\lambda_{min} = \lambda_0 < \lambda_1 < \lambda_1 < \ldots < \lambda_n < \ldots$ ([4]). At the same time the eigenvalue λ_i corresponds to the unique eigenfunction $(\psi_{0,i}, \psi_{j_c,i})$, $n = 0,1,2,\ldots$, which satisfies the condition (1.7g).

For a minimal eigenvalue $\lambda_{min} > 0$ the corresponding solution $\big(\varphi_1(x,p), \varphi_{j_c}(x,p)\big)$ of the nonlinear boundary value problem (1.4) - (1.6) is stable with respect to small space-time perturbations. For a minimal eigenvalue $\lambda_{min} < 0$, this solution is non stable. The value $\lambda_{min} = 0$ corresponding to the point of bifurcation, at which the stable solutions become unstable and vice verse.

The main physics' characteristics of the solutions of the problem (1.4) are the fool energy described by

$$F[\varphi] = \int_{-l}^{0} \left[\frac{1}{2}\varphi_{0,x}^2 + 1 - \cos\varphi_0 - \gamma\varphi\right] dx + \int_{0}^{l} \left[\frac{1}{2}\varphi_{j_c,x}^2 + j_c(1-\cos\varphi_{j_c}) - \gamma\varphi\right] dx - h_e\Delta\varphi. \tag{1.8}$$

The fool magnetic flux through the junction is described by

$$\Delta\varphi = \frac{1}{2\pi}\left[\int_{-l}^{0}\varphi_{0,x}(x)\,dx + \int_{0}^{l}\varphi_{j_c,x}(x)\,dx\right] = \frac{1}{2\pi}\left[\varphi_{j_c}(l) - \varphi_0(-l)\right], \tag{1.9}$$

and the average magnetic flux is

$$N[\varphi] = \frac{1}{2l\pi}\left[\int_{-l}^{0}\varphi_0(x)dx + \int_{0}^{l}\varphi_{j_c}(x)dx\right]. \tag{1.10}$$

The average current of interactions of distrutions of the magnetic flux in the junction with nonhomogeouty is given by

$$J_p[\varphi] = \frac{j_c - 1}{2l}\int_{0}^{l}\sin\varphi\,dx. \tag{1.11}$$

In the paper the exact analytic solutions of the system (1.4) will be obtained in the case of "infinite" 0-j_cJJ . The oneflaxons distributions in "infinite" homogeneous and j_c junctions will be obtained.

The static distributions of the magnetic flux in 0-j_cJJ for zero outer current $\gamma = 0$ will be given as a result of the nonlinear interaction between the distributions in "virtual" homogeneous and j_c junctions at point $x = 0$. From mathematical point of view this means that for any given solution of the nonlinear boundary value problem (1.4) - (1.6) we obtain the solutions of the corresponding boundary value problem (1.1).

For numerical solving of the nonlinear boundary value problem (1.4) - (1.6) a continuous analogue of Nyuton's method (CAMN) as well as the spline-collocation method will be applied. For the obtained solutions in 0-j_cJJ their corresponding distributions in homogeneous and j_c-junctions are obtained. The corresponding Cauchy problems with an additional condition are solved.

2. Main results

2.1. Semiflaxons in structures of parallel connected junctions with different currents, constructed by oneflaxons

Fix a distribution in any of both junctions with different critical currents. At the junction 0-j_cJJ , constructed by their multiply connection, new state correspond them, which are resulting on "glue" of distributions of in interacted junctions at point $x = 0$.

This method give us an opportunity to obtain the exact solutions in the case of "infinite" 0-j_cJJ junction where the conditions are from the type (1.4) for $\gamma = 0$, $h_e = 0$ and $l \to \infty$. For constructing the semiflaxons we use oneflaxons solutions in "infinite" homogeneous junction $j_c(x) \equiv 1$

$$\Phi_0^1(x) = 4 \arctan \exp \{x + \xi_1\}, \tag{2.1}$$

where ξ_1 is a coefficient of translation of the centered flaxon ($\xi_1 = 0$) and its corresponding oneflaxon solutions in "infinite" junction with an amplitude of Josephson current $j_c > 0$

$$\Phi_{j_c}^1(x) = 4 \arctan \exp \left\{ \sqrt{j_c}\, x + \xi_2 \right\}, \tag{2.2}$$

At the point $x = 0$ there are conditions for continuity of the function (1.5a) and its first derivative (1.5b). Therefore, the obtained exact analytic solutions in 0-kJJ are not twice smooth.

The obtained semiflaxons in 0-j_cJJ will be denoted by $[\Phi^1 \wedge \Phi^1]_{0j}$? $[\Phi^1 \vee \Phi^1]_{0j}$, and the corresponding their state in $j_c - 0$ junction by $[\Phi^1 \wedge \Phi^1]_{j0}$ and $[\Phi^1 \vee \Phi^1]_{j0}$.

In the case $j_c \in [0, 2)$ (with an exception of the trivial case $j_c = 1$) from the conditions for continuity it follows that in an "infinite" 0-j_cJJ there are no semiflaxons $[\Phi^1 \wedge \Phi^1]_{0j}$ constructed by oneflaxon solutions (2.1) and (2.2).

In the case $j_c \in [-2, 0)$ the nonlinear problem (1.4) for $\gamma = 0$, $h_e = 0$ and $l \to \infty$ will be written in the form

$$-\varphi_{0,xx} + \sin \varphi_0 = 0, \quad x \in (-l, 0), \tag{2.3a}$$

$$-\varphi_{j_c,xx} + (-j_c) \sin(\varphi_{j_c} + \pi) = 0, \quad x \in (0, l). \tag{2.3b}$$

Oneflaxons solutions in "infinite" junction with an amplitude of Josephson current $j_c \in [-2, 0)$ are from the type

$$\Phi_{j_c}^1(x) = 4 \arctan \exp \left\{ \sqrt{-j_c}\, x + \xi_2 \right\} - \pi, \tag{2.4}$$

The conditions (1.5a) and (1.5b) lead to the following system of equations for ξ_1 and ξ_2

$$\exp\{\xi_1\} = \frac{\exp\{\xi_2\} - 1}{1 + \exp\{\xi_2\}}, \tag{2.5a}$$

$$\frac{\exp\{\xi_2\}}{1 + \exp\{2\xi_2\}} = \sqrt{-j_c} \frac{\exp\{\xi_1\}}{1 + \exp\{2\xi_1\}}, \tag{2.5b}$$

Simplify the system (2.5) and obtain the quadratic equation

$$y^2 - 2\sqrt{-j_c}\, y - 1 = 0, \tag{2.6}$$

where $y = \exp\{\xi_2\}$. The roots of the quadratic equation are $y_{1,2} = \sqrt{-j_c} \pm \sqrt{1 - j_c}$. In order to have a solution of system, it is necessary the right side of the equation (2.5a) be positive. Therefore, the condition $\exp\{\xi_2\} \in (1, \infty)$ holds. Then, for the positive root of the quadratic equation, the solution of the system (2.5) will be

$$\xi_2 = \ln(\sqrt{-j_c} + \sqrt{1 - j_c}),$$

where the value of ξ_1 is obtained by(2.5a).

The analytic expression of the distribution in 0-j_cJJ is by the type:

$$[\Phi^1 \wedge \Phi^1]_{0j}(x) = \begin{cases} 4 \arctan \exp\{x - \xi_1\}, & x \in (-\infty, 0], \\ 4 \arctan \exp\left\{\sqrt{-j_c}x + \xi_2\right\} - \pi, & x \in [0, \infty). \end{cases} \tag{2.7}$$

The polyflaxon in $j_c - 0$ junction is $[\Phi^1 \vee \Phi^1]_{\pi 0}$:

$$[\Phi^1 \vee \Phi^1]_{j0}(x) = \begin{cases} 4 \arctan \exp\left\{\sqrt{-j_c}x + \xi_2\right\} - \pi, & x \in (-\infty, 0], \\ 4 \arctan \exp\{x - \xi_1\}, & x \in [0, \infty). \end{cases} \tag{2.8}$$

Differently than the homogeneous junction for normal fool magnetic fluxes of semiflaxons (2.7) and (2.8) we obtain:

$$\Delta[\Phi^1 \wedge \Phi^1]_{0\pi} = \frac{1}{2},$$

$$\Delta[\Phi^1 \vee \Phi^1]_{\pi 0} = \frac{3}{2}.$$

For $j_c = 1$ we obtain the case of 0-πJJ .

Let us now consider the distribution Φ_0^1 (2.1) in a homogeneous junction and a flaxon in a junction with an amplitude j_c, which is obtained by a translation of $\Phi_{j_c}^1$ (2.2) with respect to the axis y with 2π. By this way we obtain one more pair of semiflaxons in 0-j_c-junction and j_c-0-junction.

The conditions (1.5a) and (1.5b) lead to the following system of equations for ξ_1 and ξ_2

$$\exp\{\xi_1\} = \frac{\exp\{\xi_2\} + 1}{1 - \exp\{\xi_2\}}, \tag{2.9a}$$

$$\frac{\exp\{\xi_2\}}{1 + \exp\{2\xi_2\}} = \sqrt{-j_c}\frac{\exp\{\xi_1\}}{1 + \exp\{2\xi_1\}}, \tag{2.9b}$$

Simplify the system (2.9) and obtain the quadratic equation

$$y^2 + 2\sqrt{-j_c}\, y - 1 = 0, \tag{2.10}$$

where$y = \exp\{\xi_2\}$. The roots of the equation (2.10) are $y_{1,2} = -\sqrt{-j_c} \pm \sqrt{1 - j_c}$. From the condition $\exp\{\xi_2\} > 0$, we obtain the following expression for the solution of the system (2.9)

$$\xi_2 = \ln(-\sqrt{-j_c} + \sqrt{1 - j_c}),$$

where the coefficient of translation ξ_1 in this case is obtained by (2.9a).

The analytic expressions in 0-kJJ for the obtained distributions are

$$[\Phi^1 \vee \Phi^1]_{0j}(x) = \begin{cases} 4 \arctan \exp\{x + \xi_1\}, & x \in (-\infty, 0], \\ 4 \arctan \exp\left\{\sqrt{-j_c}x - \xi_2\right\} + \pi, & x \in [0, \infty). \end{cases} \tag{2.11}$$

The semiflaxon in k-0-junction is $[\Phi^1 \vee \Phi^1]_{\pi 0}$ and

$$[\Phi^1 \wedge \Phi^1]_{j0}(x) = \begin{cases} 4 \arctan \exp\left\{\sqrt{-j_c}x - \xi_2\right\} + \pi, & x \in (-\infty, 0], \\ 4 \arctan \exp\{x + \xi_1\}, & x \in [0, \infty). \end{cases} \tag{2.12}$$

2.2. Representation of semiflaxons in $0-j_c$ junctions as a result of flaxons in homogeneous and j_c junctions

We will consider the static distributions of magnetic flux in finite 0-j_cJJ for zero magnetic field $\gamma = 0$, as a result of a nonlinear interaction of the distributions in “virtual” homogeneous and a junction with an amplitude of Josephson current j_c. For this purpose we will formulate the conditions that will allow us to present any solution of the nonlinear boundary value problem (1.4) - (1.6) as a result of the interactions between the solutions of the boundary value problem (1.1) in “virtual” homogeneous and j_c junctions with a length different than the length of the initial junction.

Indeed, let $(\varphi_0, \varphi_{j_c})$ is a solution of the nonlinear boundary value problem (1.4) - (1.6) in 0-j_cJJ , where $\varphi_0(x)$ is the solution from the left, i.e. in homogeneous part, and $\varphi_{j_c}(x)$ — is in the right part, i.e. in j_c half. Then the conditions for continuity has the form:

$$\varphi_0(0) = \varphi_{j_c}(0), \tag{2.13a}$$

$$\varphi_{0,x}(0) = \varphi_{j_c,x}(0). \tag{2.13b}$$

The function $\varphi_0(x)$, defined on $(-l, 0)$, is a solution of the equation (1.1a) with boundary conditions (1.6a) and (2.13b), and satisfies the additional condition

(2.13a). To find the solution of (1.1) in a homogeneous junction, which takes part in the construction of (φ_0, φ_π), we obtain a solution $\phi_0(x)$ of the equation (1.1a) for $j_c = 1$, which satisfies the following conditions

$$\phi_0(0) = \varphi_0(0), \quad \phi_{0,x}(0) = \varphi_{0,x}(0),$$

$$\phi_{0,x}(l_0) = h_e,$$

where l_0 is an unknown constant.

The function $\varphi_{j_c}(x)$, defined on $(0, l)$, is a solution of the equation (1.1a) with boundary conditions (1.6b) and (2.13b), and satisfies the additional condition (2.13a). In this case to obtain the solution in j_c junction we need the solution $\phi_{j_c}(x)$ of the equation (1.1), for which the conditions

$$\phi_{j_c}(0) = \varphi_{j_c}(0), \quad \phi_{j_c,x}(0) = \varphi_{j_c,x}(0),$$

$$\phi_{j_c,x}(-l_{j_c}) = h_e,$$

hold, where l_{j_c} is an unknown constant.

By this way the problem for finding solutions in homogeneous and j_c junctions is reduced to Stephan's problem with unknown right (or left) bound. For solving of the problem, there are two different approaches.

Approach 1. Cauchy problem with an additional condition.

For obtaining the function $\phi_0(x)$ we solve the following initial value problem

$$-\phi_{0,xx} + \sin\phi_0 = 0, \quad x \in (0, l_0), \tag{2.14a}$$

$$\phi_0(0) = \varphi_0(0), \tag{2.14b}$$

$$\phi_{0,x}(0) = \varphi_{0,x}(0), \tag{2.14c}$$

with an additional condition $\phi_{0,x}(l_0) = h_e$, where l_0 is an unknown constant.

The function $\phi_{j_c}(x)$ is a solution of the following initial value problem

$$-\phi_{j_c,xx} + j_c \sin\phi_{j_c} = 0, \quad x \in (-l_\pi, 0), \tag{2.15a}$$

$$\phi_{j_c}(0) = \varphi_{j_c}(0), \tag{2.15b}$$

$$\phi_{j_c,x}(0) = \varphi_{j_c,x}(0), \tag{2.15c}$$

with an additional condition $\phi_{j_c,x}(-l_k) = h_e$, where l_{j_c} is an unknown constant.

Approach 2. Nonlinear problem with eigenvalues.

In the homogeneous junction the function $\phi_0(x)$ could be obtained as a solution of the problem for eigenvalues

(2.16a) $$\phi_0(0) = \varphi_0(0),$$

(2.16b) $$\phi_{0,x}(0) = \varphi_{0,x}(0),$$

(2.16c) $$-\phi_{0,xx} + \sin\phi_0 = 0, \quad x \in (0, l_0),$$

(2.16d) $$\phi_{0,x}(l_0) = h_e,$$

where l_0 is an unknown constant.

The function $\phi_{j_c}(x)$ is a solution of the following problem for eigenvalues:

(2.17a) $$\phi_{j_c}(0) = \varphi_{j_c}(0)$$

(2.17b) $$\phi_{j_c,x}(0) = \varphi_{j_c,x}(0),$$

(2.17c) $$\phi_{j_c,xx} + j_c \sin\phi_{j_c} = 0, \quad x \in (-l_\pi, 0),$$

(2.17d) $$\phi_{j_c,x}(-l_\pi) = h_e,$$

where l_{j_c} is an unknown constant.

If we obtain functions $\phi_0(x)$ and $\phi_{j_c}(x)$ by one of the above described approaches, then we could solve the set up Stephan's problems.

So, we construct the function $\Phi_0(x)$ by

$$\Phi_0(x) = \begin{cases} \varphi_0(x), & x \in (-l, 0] \\ \phi_0(x), & x \in (0, l_0] \end{cases}$$

which is a solution of the problem:

(2.18a) $$-\Phi_{xx} + \sin\Phi = 0, \quad x \in (-l, l_0),$$

(2.18b) $$\Phi_x(-l) = h_e,$$

(2.18c) $$\Phi_x(l_0) = h_e,$$

Analogously, we define a function $\Phi_{j_c}(x)$ by

$$\Phi_{j_c}(x) = \begin{cases} \phi_{j_c}(x) & x \in (-l_{j_c}, 0] \\ \varphi_{j_c}(x) & x \in (0, l] \end{cases}$$

which is a solution of the problem:

(2.19a) $$-\Phi_{xx} + j_c \sin\Phi = 0, \quad x \in (-l_{j_c}, l),$$

(2.19b) $$\Phi_x(-l_\pi) = h_e,$$

(2.19c) $$\Phi_x(l) = h_e.$$

The solution $(\varphi_0, \varphi_{j_c})$ of the nonlinear boundary value problem (1.4) — (1.6) in 0-j_cJJ is a result of the interaction of the solution $\Phi_0(x)$ of the problem (2.18) and the solution $\Phi_{j_c}(x)$ of the problem (2.19). We note that all three boundary value problems are defined on different intervals.

3. Numerical experiment

We will consider the static distributions of the magnetic flux in 0-j_cJJ for zero outer current ($\gamma = 0$), resulting of the nonlinear interaction of the distributions in "virtual" homogeneous and j_c-junctions at point $x = 0$. From mathematical point of view this means that for any given solution of the nonlinear boundary value problem (1.4) - (1.6) we obtain the solutions of the boundary value problem (2.18) and (2.19).

For numerical solving of the nonlinear boundary value problem (1.4) - (1.6) a continuous analogue of Nyuton's method (CAMN) as well as the spline-collocation method will be applied. For the obtained solutions in 0-πJJ their corresponding distributions in homogeneous and π-junctions are obtained. The corresponding Cauchy problems with an additional condition are solved. We will study the influence of the outer magnetic flux on the basic stable semiflaxons in 0-πJJ and the corresponding "virtual" junctions.

For zero outer current ($\gamma = 0$), the solutions of homogeneous and j_c junctions are presented by elleptic functions and therefore, the Cauchy problems with additional conditions (2.14) and (2.15) has countable set of solutions. Consider the distributions of the magnetic flux in vertual homogeneous and j_c junctions, which correspond to the initial semiflaxon in 0-j_cJJ . The solutions which are determined by them, depend on the values oft the basic numerical characteristics - the functional of fool energy, thefool magnetic flux and the average magnetic flux.

At 0-j_cJJ with length $2l = 7$ we will consider the basic semiflaxon. Denote it by $S^{1,1} = \Phi^1 \wedge \Phi^1$, where in virtual homogeneous and j_c junctions we have oneflaxons solutions.

On fig. 3.1 by the continuous curve (a, a) is graphed the semiflaxon $S^{1,1}$ for zero outer magnetic field $h_e = 0$ and amplitude of Josephson current $j_c = -0.5$. Oneflaxons in virtual homogeneous and j_c junctions are graphed by the curves (a, b) and (b, a). Note that oneflaxons are solutions of the nonlinear boundary value problems (2.18) and (2.19).

The inner magnetic field of $S^{1,1}$ and the oneflaxons for $j_c = -0.5$ are graphed on fig. 3.2.

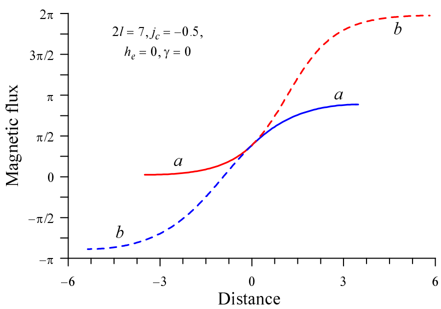

Figure 3.1: Semiflaxon $S^{1,1}$ for $j_c = -0.5$

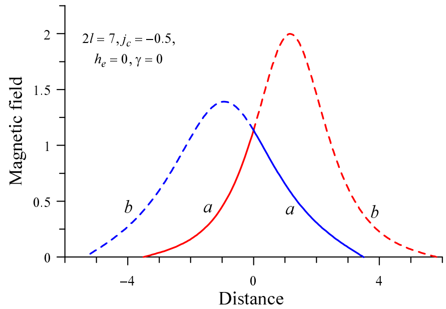

Figure 3.2: Inner magnetic field of $S^{1,1}$ for $j_c = -0.5$

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КОНСТРУИРАНЕ НА ФЛАКСОНИ В ДЖОЗЕФСОНОВИ КОНТАКТИ

Тодор Бояджиев, Христо Мелемов

Резюме. В настоящата статия се изследват аналитично и числено статичните разпределения на магнитния поток в двойка последователно свързани Джозефсонови контакти с еднаква дължина и различна амплитуда на Джозефсоновия ток 0-j_cJJ . Частен случай на тези структури са $0 - \pi$ контактите. Получени са аналитични решения на разпределения на магнитния поток в "безкраен" 0-j_cJJ с помощта на еднофлаксонните разпределения в хомогенен и j_c контакт.

Получените решения на модела за тези структури са $\mathcal{C}^1$-гладки. В статията разпределенията на магнитния поток се разглеждат като резултат от нелинейното взаимодействие на флаксони в двата субконтакта в общата им граница.