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THRIAD COMPOSITIONS IN AFFINELY CONNECTED SPACES A_{3m}

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Abstract. Let the affinely connected space without a torsion A_{3m} be a space of a composition $(X_{2m} \times X_m)$ generated by the affinor a_α^β. Another two affinors b_α^β and c_α^β related with a_α^β are introduced. Two new compositions $(Y_{2m} \times Y_m)$ and $(Z_{2m} \times Z_m)$ generated by the affinors b_α^β and c_α^β are considered and so it is determined a thriad compositions.

It is proved that if two of the thriad compositions are special of the type $(d-d)$ or $(ch-ch)$, then the third composition is of the same type. The characteristics of the special triad compositions are found. The kind of the spaces A_{3m} which contain special thriad compositions is defined in the adapted with the composition $(X_{2m} \times X_m)$ coordinate system. It is prove that if in Weyl space W_{3m} there is a thriad compositions of type $(ch-ch)$, then this space is Riemannian.

The topological product of the manifolds $X_m \times Y_m \times Z_m$ is called a bundle and it is noticed S_3^{mmm}. The characteristics of the special bundles $S_3^{mmm} \in A_{3m}$ are found and it is defined the kind of the spaces A_{3m} containing such special bundles.

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1. Preliminary

Let A_N be a space with a symmetric affinely connection, defined by $\Gamma_{\alpha\beta}^\sigma$. Consider one composition $(X_n \times X_m), n+m=N$ in A_N of two base differentiable manifolds X_n and X_m, i.e. their topological product. Two positions

$P(X_n)$ and $P(X_m)$ of the base manifolds pass through any point of the space $A_N(X_n \times X_m)$.

We shall use besides an arbitrary coordinate system $x^\alpha (\alpha = 1, 2, \ldots, n+m)$ and the coordinate system $(u^i, u^{\bar{i}})$ $(i = 1, 2, \ldots, n;\ \bar{i} = n+1, n+2, \ldots, n+m)$ which is introduced by Norden [2]. This coordinate system is called adapted with the composition.

According to [2], [3] the giving of the field of the affinor a_α^β satisfying the condition

$$a_\alpha^\sigma a_\sigma^\beta = \delta_\alpha^\beta \tag{1}$$

is equivalent to the giving of the composition $(X_n \times X_m), n + m = N$ in A_N. The affinor a_α^β is called an affinor of the composition.

According to [3], [5] the condition for integrability of the structure characterizes with the equality

$$a_\beta^\sigma \nabla_{[\alpha}\ a_{\sigma]}^\nu - a_\alpha^\sigma \nabla_{[\beta}\ a_{\sigma]}^\nu = 0. \tag{2}$$

The projecting affinors $\overset{n}{a}{}_\alpha^\beta$ *and* $\overset{m}{a}{}_\alpha^\beta$ [3], [4] define by the equalities

$$\overset{n}{a}{}_\alpha^\beta = \frac{1}{2}(\delta_\alpha^\beta + a_\alpha^\beta)\ ,\quad \overset{m}{a}{}_\alpha^\beta = \frac{1}{2}(\delta_\alpha^\beta - a_\alpha^\beta)\ , \tag{3}$$

satisfy the following conditions [4]:

$$\begin{aligned} &\overset{n}{a}{}_\alpha^\beta + \overset{m}{a}{}_\alpha^\beta = \delta_\alpha^\beta\ ,\quad \overset{n}{a}{}_\alpha^\beta - \overset{m}{a}{}_\alpha^\beta = a_\alpha^\beta\ ,\\ &\overset{n}{a}{}_\alpha^\beta \overset{n}{a}{}_\beta^\gamma = \overset{n}{a}{}_\alpha^\gamma\ ,\quad \overset{m}{a}{}_\alpha^\beta \overset{m}{a}{}_\beta^\gamma = \overset{m}{a}{}_\alpha^\gamma\ ,\quad \overset{m}{a}{}_\alpha^\beta \overset{n}{a}{}_\beta^\gamma = 0\ . \end{aligned} \tag{4}$$

Any vector $v^\alpha \in A_N$ has the representation [4]

$$v^\alpha = \overset{n}{a}{}_\sigma^\alpha\ v^\sigma + \overset{m}{a}{}_\sigma^\alpha\ v^\sigma = \overset{n}{V}{}^\alpha + \overset{m}{V}{}^\alpha, \tag{5}$$

where $\overset{n}{V}{}^\alpha = \overset{n}{a}{}_\sigma^\alpha v^\sigma \in P(X_n)$, $\overset{m}{V}{}^\alpha = \overset{m}{a}{}_\sigma^\alpha v^\sigma \in P(X_m)$.

The matrices of the affinors a_α^β, $\overset{n}{a}{}_\alpha^\sigma$, $\overset{m}{a}{}_\alpha^\sigma$ in the adapted with the composition coordinate system $(u^i, u^{\bar{i}})$ have the following representation [3], [4]

$$(a_\alpha^\beta) = \begin{pmatrix} \delta_j^i & 0 \\ 0 & -\delta_{\bar{j}}^{\bar{i}} \end{pmatrix},\ (\overset{n}{a}{}_\alpha^\beta) = \begin{pmatrix} \delta_i^j & 0 \\ 0 & 0 \end{pmatrix},\ (\overset{m}{a}{}_\alpha^\beta) = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{\bar{j}}^{\bar{i}} \end{pmatrix}. \tag{6}$$

The tensors

$$A^{\sigma}_{\alpha\beta} = \nabla_{\alpha} a^{\sigma}_{\beta} \ , \quad \overset{n}{A}{}^{\sigma}_{\alpha\beta} = \nabla_{\alpha} \overset{n}{a}{}^{\sigma}_{\beta} \ , \quad \overset{m}{A}{}^{\sigma}_{\alpha\beta} = \nabla_{\alpha} \overset{m}{a}{}^{\sigma}_{\beta} \tag{7}$$

are introduced in [3] and for them the equalities

$$\begin{gathered} A^{s}_{ij} = A^{s}_{\bar{i}j} = A^{\bar{s}}_{i\bar{j}} = A^{\bar{s}}_{\bar{i}\bar{j}} = 0 \ , \\ A^{s}_{i\bar{j}} = -2\Gamma^{s}_{i\bar{j}} \ , \ A^{s}_{\bar{i}\bar{j}} = -2\Gamma^{s}_{\bar{i}\bar{j}} \ , \ A^{\bar{s}}_{ij} = 2\Gamma^{\bar{s}}_{ij} \ , \ A^{\bar{s}}_{\bar{i}j} = 2\Gamma^{\bar{s}}_{\bar{i}j} \ . \end{gathered} \tag{8}$$

are fulfilled. Norden and Timofeev obtain in [3] the characteristics of the following special compositions:

$(d{-}d)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along any line of the space characterize with the condition

$$A^{\sigma}_{\alpha\beta} = 0 \tag{9}$$

and in the adapted with the composition coordinate system - with the equalities

$$\Gamma^{k}_{\bar{i}\alpha} = \Gamma^{\bar{k}}_{i\alpha} = 0. \tag{10}$$

$(ch{-}ch)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along the lines of X_m and X_n, respectively characterize with the condition

$$A^{\sigma}_{[\alpha\beta]} = 0 \tag{11}$$

and in the adapted with the composition coordinate system - with the equalities

$$\Gamma^{\bar{k}}_{\bar{i}j} = \Gamma^{k}_{i\bar{j}} = 0. \tag{12}$$

$(g{-}g)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along the lines of X_n and X_m, respectively characterize with the condition

$$a^{\sigma}_{\alpha} \ A^{\nu}_{\beta\sigma} + a^{\sigma}_{\beta} \ A^{\nu}_{\sigma\alpha} = 0 \tag{13}$$

and in the adapted with the composition coordinate system - with the equalities

$$\Gamma^{\bar{k}}_{ij} = \Gamma^{k}_{\bar{i}\bar{j}} = 0. \tag{14}$$

$(ch{-}X_m)$-compositions for which the positions $P(X_n)$ are parallelly translated along the lines of X_m characterize with the condition

$$\overset{m}{a}{}^{\sigma}_{\alpha}\,\overset{n}{a}{}^{\nu}_{\beta}\,\overset{n}{A}{}^{\delta}_{\sigma\nu}=0 \tag{15}$$

and in the adapted with the composition coordinate system - with the equalities

$$\Gamma^{\bar{k}}_{\bar{i}\,j}=0. \tag{16}$$

$(X_n{-}ch)$-compositions for which the positions $P(X_m)$ are parallelly translated along the lines of X_n characterize with the condition

$$\overset{n}{a}{}^{\sigma}_{\alpha}\,\overset{m}{a}{}^{\nu}_{\beta}\,\overset{m}{A}{}^{\delta}_{\sigma\nu}=0 \tag{17}$$

and in the adapted with the composition coordinate system - with the equalities

$$\Gamma^{k}_{i\,\bar{j}}=0. \tag{18}$$

$(g{-}X_m)$-compositions for which the positions $P(X_n)$ are parallelly translated along the lines of X_n characterize with the condition

$$\overset{n}{a}{}^{\sigma}_{\alpha}\,\overset{n}{a}{}^{\nu}_{\beta}\,\overset{n}{A}{}^{\delta}_{\sigma\nu}=0 \tag{19}$$

and in the adapted with the composition coordinate system - with the equalities

$$\Gamma^{\bar{k}}_{ij}=0. \tag{20}$$

$(X_n{-}g)$-compositions for which the positions $P(X_m)$ are parallelly translated along the lines of X_m characterize with the condition

$$\overset{m}{a}{}^{\sigma}_{\alpha}\,\overset{m}{a}{}^{\nu}_{\beta}\,\overset{m}{A}{}^{\delta}_{\sigma\nu}=0 \tag{21}$$

and in the adapted with the composition coordinate system - with the equalities

$$\Gamma^{k}_{\bar{i}\,\bar{j}}=0. \tag{22}$$

2. A thriad compositions in $A_{3m}(X_{2m} \times X_m)$

Let an affinor a_α^β satisfying the condition (1) be given in the space A_{3m} $(m \geq 1)$ with symmetric connection. If $\underset{1}{v}^\sigma, \underset{2}{v}^\sigma, \dots, \underset{3m}{v}^\sigma$ are eigen-vectors of the matrix (a_α^β) and

$$a_\alpha^\sigma \underset{\rho}{v}^\alpha = \underset{\rho}{v}^\sigma (\rho = 1, 2, \dots, 2m), \quad a_\alpha^\sigma \underset{\rho}{v}^\alpha = -\underset{\rho}{v}^\sigma (\rho = 2m+1, 2m+2, \dots, 3m),$$

then the affinor a_α^β defines the composition $(X_{2m} \times X_m) \in A_{3m}$.

Introduce the following indices

$$\begin{gathered} i, j, k, l = 1, 2, \dots, m; \quad p, q, r, s = m+1, m+2, \dots, 2m; \\ a, b, c, d = 2m+1, 2m+2, \dots, 3m; \\ \alpha, \beta, \gamma, \delta, \sigma, \rho, \nu = 1, 2, \dots, m, \dots, 2m, \dots, 3m. \end{gathered} \tag{23}$$

According to (23) in the adapted with the composition $(X_{2m} \times X_m)$ coordinate system we can write

$$(a_\alpha^\beta) = \begin{pmatrix} \delta_i^j & 0 & 0 \\ 0 & \delta_p^q & 0 \\ 0 & 0 & -\delta_a^b \end{pmatrix}. \tag{24}$$

With the help of the equalities

$$\begin{aligned} b_\alpha^\sigma \underset{\rho}{v}^\alpha &= -\underset{\rho}{v}^\sigma \quad \text{when } \rho = 1, 2, \dots, m; \\ b_\alpha^\sigma \underset{\rho}{v}^\alpha &= \underset{\rho}{v}^\sigma \quad \text{when } \rho = m+1, m+2, \dots, 2m, 2m+1, \dots, 3m; \\ c_\alpha^\sigma \underset{\rho}{v}^\alpha &= \underset{\rho}{v}^\sigma \quad \text{when } \rho = 1, 2, \dots, m; \ 2m+1, 2m+2, \dots, 3m \\ c_\alpha^\sigma \underset{\rho}{v}^\alpha &= -\underset{\rho}{v}^\sigma \quad \text{when } \rho = m+1, m+2, \dots, 2m, \end{aligned} \tag{25}$$

we define the affinors b_α^σ and c_α^σ. Obviously the affinors b_α^σ and c_α^σ satisfy the equality (1). From (25) it follows that in the adapted with the composition $(X_{2m} \times X_m)$ coordinate system the matrices of these affinors have the form

$$(b_\alpha^\beta) = \begin{pmatrix} -\delta_i^j & 0 & 0 \\ 0 & \delta_p^q & 0 \\ 0 & 0 & \delta_a^b \end{pmatrix}, \quad (c_\alpha^\beta) = \begin{pmatrix} \delta_i^j & 0 & 0 \\ 0 & -\delta_p^q & 0 \\ 0 & 0 & \delta_a^b \end{pmatrix}. \tag{26}$$

The compositions $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ generated from the affinors a_α^β, b_α^β, c_α^β, respectively we call a thriad compositions in the space A_{3m}.

Let notice the projecting affinors of the compositions $(Y_{2m} \times Y_m)$ and $(Z_{2m} \times Z_m)$ by $\overset{n}{b}{}^{\alpha}_{\sigma}$, $\overset{m}{b}{}^{\alpha}_{\sigma}$ and $\overset{n}{c}{}^{\alpha}_{\sigma}$, $\overset{m}{c}{}^{\alpha}_{\sigma}$ respectevly. According to (5) for an arbitrary vector v^α

$$
\begin{aligned}
v^\alpha &= \overset{n}{a}{}^{\alpha}_{\sigma}\, v^\sigma + \overset{m}{a}{}^{\alpha}_{\sigma}\, v^\sigma = \overset{n}{V}{}^{\alpha} + \overset{m}{V}{}^{\alpha},\\
v^\alpha &= \overset{n}{b}{}^{\alpha}_{\sigma}\, v^\sigma + \overset{m}{b}{}^{\alpha}_{\sigma}\, v^\sigma = \underset{1}{\overset{n}{V}}{}^{\alpha} + \underset{1}{\overset{m}{V}}{}^{\alpha},\\
v^\alpha &= \overset{n}{c}{}^{\alpha}_{\sigma}\, v^\sigma + \overset{m}{c}{}^{\alpha}_{\sigma}\, v^\sigma = \underset{2}{\overset{n}{V}}{}^{\alpha} + \underset{2}{\overset{m}{V}}{}^{\alpha},
\end{aligned}
\tag{27}
$$

where

$$
\begin{gathered}
\overset{n}{V}{}^{\alpha} \in P(X_{2m}),\ \overset{m}{V}{}^{\alpha} \in P(X_m)\ ;\\
\underset{1}{\overset{n}{V}}{}^{\alpha} \in P(Y_{2m}),\ \underset{1}{\overset{m}{V}}{}^{\alpha} \in P(Y_m)\ ;\ \ \underset{2}{\overset{n}{V}}{}^{\alpha} \in P(Z_{2m}),\ \underset{2}{\overset{m}{V}}{}^{\alpha} \in P(Z_m).
\end{gathered}
\tag{28}
$$

Because of (3) and (4) for the projecting affinors $\overset{n}{b}{}^{\alpha}_{\sigma}$, $\overset{m}{b}{}^{\alpha}_{\sigma}$ and $\overset{n}{c}{}^{\alpha}_{\sigma}$, $\overset{n}{c}{}^{\alpha}_{\sigma}$ we have

$$
\begin{array}{llll}
\overset{n}{b}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^\beta_\alpha + b^\beta_\alpha), & \overset{m}{b}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^\beta_\alpha - b^\beta_\alpha), & \overset{n}{b}{}^{\beta}_{\alpha} + \overset{m}{b}{}^{\beta}_{\alpha} = \delta^\beta_\alpha, & \overset{n}{b}{}^{\beta}_{\alpha} - \overset{m}{b}{}^{\beta}_{\alpha} = b^\beta_\alpha,\\
\overset{n}{c}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^\beta_\alpha + c^\beta_\alpha), & \overset{m}{c}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^\beta_\alpha - c^\beta_\alpha), & \overset{n}{c}{}^{\beta}_{\alpha} + \overset{m}{c}{}^{\beta}_{\alpha} = \delta^\beta_\alpha, & \overset{n}{c}{}^{\beta}_{\alpha} - \overset{m}{c}{}^{\beta}_{\alpha} = c^\beta_\alpha,\\
\overset{n}{b}{}^{\beta}_{\alpha}\, \overset{n}{b}{}^{\gamma}_{\beta} = \overset{n}{b}{}^{\gamma}_{\alpha}, & \overset{m}{b}{}^{\beta}_{\alpha}\, \overset{m}{b}{}^{\gamma}_{\beta} = \overset{m}{b}{}^{\gamma}_{\alpha}, & \overset{m}{b}{}^{\beta}_{\alpha}\, \overset{n}{b}{}^{\gamma}_{\beta} = 0, & \overset{n}{b}{}^{\beta}_{\alpha}\, \overset{m}{b}{}^{\gamma}_{\beta} = 0,\\
\overset{n}{c}{}^{\beta}_{\alpha}\, \overset{n}{c}{}^{\gamma}_{\beta} = \overset{n}{c}{}^{\gamma}_{\alpha}, & \overset{m}{c}{}^{\beta}_{\alpha}\, \overset{m}{c}{}^{\gamma}_{\beta} = \overset{m}{c}{}^{\gamma}_{\alpha}, & \overset{m}{c}{}^{\beta}_{\alpha}\, \overset{n}{c}{}^{\gamma}_{\beta} = 0, & \overset{n}{c}{}^{\beta}_{\alpha}\, \overset{m}{c}{}^{\gamma}_{\beta} = 0.
\end{array}
\tag{29}
$$

Following [3] we introduce the tensors

$$
\begin{aligned}
B^\sigma_{\alpha\beta} &= \nabla_\alpha b^\sigma_\beta\ ,\ \ \overset{n}{B}{}^{\sigma}_{\alpha\beta} = \nabla_\alpha\, \overset{n}{b}{}^{\sigma}_{\beta}\ ,\ \ \overset{m}{B}{}^{\sigma}_{\alpha\beta} = \nabla_\alpha\, \overset{m}{b}{}^{\sigma}_{\beta}\ ;\\
C^\sigma_{\alpha\beta} &= \nabla_\alpha c^\sigma_\beta\ ,\ \ \overset{n}{C}{}^{\sigma}_{\alpha\beta} = \nabla_\alpha\, \overset{n}{c}{}^{\sigma}_{\beta}\ ,\ \ \overset{m}{C}{}^{\sigma}_{\alpha\beta} = \nabla_\alpha\, \overset{m}{c}{}^{\sigma}_{\beta}\ .
\end{aligned}
\tag{30}
$$

Theorem 1. *In the adapted with the composition* $(X_{2m}\times X_m)$ *coordinate system the components of the tensors* $A^{\sigma}_{\alpha\beta},\ B^{\sigma}_{\alpha\beta},\ C^{\sigma}_{\alpha\beta}$ *satisfy the equalities*

$$
\begin{aligned}
&A^l_{ij}=A^l_{ip}=A^l_{pi}=A^l_{pq}=A^p_{ij}=A^p_{iq}=A^p_{qi}=A^p_{qs}=0\ ,\\
&A^l_{ai}=A^l_{ap}=A^p_{ai}=A^p_{aq}=A^a_{ib}=A^a_{pb}=A^a_{bc}=0\ ;\\
&A^l_{ia}=-2\Gamma^l_{ia},\ A^l_{pa}=-2\Gamma^l_{pa},\ A^p_{ia}=-2\Gamma^p_{ia},\ A^p_{qa}=-2\Gamma^p_{qa},\\
&A^l_{ab}=-2\Gamma^l_{ia},\ A^p_{ab}=-2\Gamma^p_{ab},\ A^a_{ij}=2\Gamma^a_{ij},\ A^a_{ip}=2\Gamma^a_{ip},\\
&A^a_{pi}=2\Gamma^a_{pi},\ A^a_{pq}=2\Gamma^a_{pq},\ A^a_{bi}=2\Gamma^a_{bi},\ A^a_{bp}=2\Gamma^a_{bp};\\
&B^l_{ij}=B^l_{si}=B^l_{ai}=B^p_{iq}=B^p_{sq}=B^p_{la}=B^p_{ab}=B^p_{sa}=0,\\
&B^p_{as}=B^c_{ls}=B^c_{sp}=B^c_{ia}=B^c_{ab}=B^c_{pa}=B^c_{ap}=0;\\
(31)\qquad &B^l_{is}=2\Gamma^l_{is},\ B^l_{ia}=2\Gamma^l_{ia},\ B^l_{sa}=2\Gamma^l_{sa},\ B^l_{as}=2\Gamma^l_{as},\ B^l_{sp}=2\Gamma^l_{sp},\\
&B^l_{ab}=2\Gamma^l_{ab},\ B^p_{ij}=-2\Gamma^p_{ij},\ B^p_{qi}=-2\Gamma^p_{qi},\ B^p_{al}=-2\Gamma^p_{al},\\
&B^c_{ij}=-2\Gamma^c_{ij},\ B^c_{sl}=-2\Gamma^c_{sl},\ B^c_{ai}=-2\Gamma^c_{ai};\\
&C^l_{ij}=C^l_{si}=C^l_{ai}=C^l_{ia}=C^l_{as}=C^l_{sa}=C^p_{iq}=C^p_{sq}=0,\\
&C^p_{as}=C^c_{ij}=C^c_{sl}=C^c_{ia}=C^c_{ai}=C^c_{ab}=C^c_{pa}=0;\\
&C^l_{ip}=-2\Gamma^l_{ip},\ C^l_{sp}=-2\Gamma^l_{sp},\ C^l_{ap}=-2\Gamma^l_{ap},\ C^l_{qi}=2\Gamma^p_{qi},\\
&C^p_{al}=2\Gamma^p_{al},\ C^p_{la}=2\Gamma^p_{la},\ C^p_{ab}=2\Gamma^p_{ab},\ C^p_{qa}=2\Gamma^p_{qa},\\
&C^p_{ij}=2\Gamma^p_{ij},\ C^c_{lp}=-2\Gamma^c_{lp},\ C^c_{pq}=-2\Gamma^c_{pq},\ C^c_{ap}=-2\Gamma^c_{ap}.
\end{aligned}
$$

Proof. From (8), taking into account (23) we obtain all equalities about the components of tensor $A^{\sigma}_{\alpha\beta}$.

Notice that in the adapted coordinate system we have

$$B^{\beta}_{\sigma\alpha}=\nabla_{\sigma}b^{\beta}_{\alpha}=\partial_{\sigma}b^{\beta}_{\alpha}-\Gamma^{\nu}_{\sigma\alpha}b^{\beta}_{\nu}+\Gamma^{\beta}_{\sigma\nu}b^{\nu}_{\alpha}=-\Gamma^{\nu}_{\sigma\alpha}b^{\beta}_{\nu}+\Gamma^{\beta}_{\sigma\nu}b^{\nu}_{\alpha}=0,$$

and

$$C^{\beta}_{\sigma\alpha}=\nabla_{\sigma}c^{\beta}_{\alpha}=\partial_{\sigma}c^{\beta}_{\alpha}-\Gamma^{\nu}_{\sigma\alpha}c^{\beta}_{\nu}+\Gamma^{\beta}_{\sigma\nu}c^{\nu}_{\alpha}=-\Gamma^{\nu}_{\sigma\alpha}c^{\beta}_{\nu}+\Gamma^{\beta}_{\sigma\nu}c^{\nu}_{\alpha}=0.$$

Using the last equalities and (26) we establish the validity of the rest equalities of (31).

Theorem 2. *The affinors* $a^\sigma_\alpha,\ b^\sigma_\alpha,\ c^\sigma_\alpha,\ \overset{n}{a}{}^\sigma_\alpha,\ \overset{m}{a}{}^\sigma_\alpha,\ \overset{n}{b}{}^\sigma_\alpha,\ \overset{m}{b}{}^\sigma_\alpha,\ \overset{n}{c}{}^\sigma_\alpha,\ \overset{m}{c}{}^\sigma_\alpha$ *satisfy*

$$a^\sigma_\alpha + b^\sigma_\alpha + c^\sigma_\alpha = \delta^\sigma_\alpha,\quad \overset{m}{a}{}^\sigma_\alpha + \overset{m}{b}{}^\sigma_\alpha + \overset{m}{c}{}^\sigma_\alpha = \delta^\sigma_\alpha,\quad \overset{n}{a}{}^\sigma_\alpha + \overset{n}{b}{}^\sigma_\alpha + \overset{n}{c}{}^\sigma_\alpha = 2\delta^\sigma_\alpha. \tag{32}$$

Proof. The proof holds in the adapted with the composition $(X_{2m} \times X_m)$ coordinate system. The first equality of (32) follows immediately from (24) and (26). Then using this result, (3) and (29) we prove the rest two equalities of (32).

Corollary 1. *If two of the compositions of the thriad compositions are* $(d - d)$ *compositions, then and the third composition is of the same type.*

Proof. Because of (32) we can write $\nabla_\beta a^\sigma_\alpha + \nabla_\beta b^\sigma_\alpha + \nabla_\beta c^\sigma_\alpha = 0$, which according to (7) and (30) takes the form $A^\sigma_{\beta\alpha} + B^\sigma_{\beta\alpha} + C^\sigma_{\beta\alpha} = 0$.

It is known that the compositions $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ are of the type $(d - d)$ if and only if $A^\sigma_{\beta\alpha} = 0,\ B^\sigma_{\beta\alpha} = 0,\ C^\sigma_{\beta\alpha} = 0$. Now the validity of the Corrolary 1 is obvious.

Corollary 2. *If two of the compositions of the thriad compositions are* $(ch - ch)$ *compositions, then and the third composition is of the same type.*

Proof. Because of (32) we can write $\nabla_{[\beta} a^\sigma_{\alpha]} + \nabla_{[\beta} b^\sigma_{\alpha]} + \nabla_{[\beta} c^\sigma_{\alpha]} = 0$, which according to (7) and (30) takes the form $A^\sigma_{[\beta\alpha]} + B^\sigma_{[\beta\alpha]} + C^\sigma_{[\beta\alpha]} = 0$.

It is known that the compositions $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ are of the type $(ch - ch)$ if and only if $A^\sigma_{[\beta\alpha]} = 0,\ B^\sigma_{[\beta\alpha]} = 0,\ C^\sigma_{[\beta\alpha]} = 0$. Now the validity of the Corrolary 2 is obvious.

3. Special thriads compositions in A_{3m}

3.1. Let the compositions $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ be $(ch - ch)$ compositions in the affinely connected space A_{3m} with a symmetric connection.

Theorem 3. *Any composition of the thriad compositions* $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ *is* $(ch - ch)$ *composition if and only if in the adapted with the composition* $(X_{2m} \times X_m)$ *coordinate system*

$$\Gamma^l_{ia} = \Gamma^l_{pa} = \Gamma^l_{ip} = \Gamma^p_{ia} = \Gamma^p_{iq} = \Gamma^p_{qa} = \Gamma^c_{ia} = \Gamma^c_{ip} = \Gamma^c_{pa} = 0. \tag{33}$$

Proof. To prove the theorem we use that the connection is symmetric, Corrolary 2 and the equalities (31) , $A^{\sigma}_{[\beta\alpha]} = 0$, $B^{\sigma}_{[\beta\alpha]} = 0$, $C^{\sigma}_{[\beta\alpha]} = 0$. So, since $A^l_{ia} = -2\Gamma^l_{ia}$, then $A^l_{[ia]} = -2\Gamma^l_{[ia]} = 0$. But $A^l_{ai} = 0$. Hence and $A^l_{ia} = 0$. From $B^c_{[ai]} = 0$ and $B^c_{ia} = 0$ we obtain $\Gamma^c_{ai} = 0$. The proof for the rest coefficients makes by analogy.

Let consider Weyl space $W_{3m}(g_{\alpha\beta}, \omega_\sigma)$ with a fundamental tensor $g_{\alpha\beta}$ and a complementary vector ω_σ which according to [1] satisfy

$$\nabla_\sigma g_{\alpha\beta} = 2\omega_\sigma g_{\alpha\beta}. \tag{34}$$

Let W_{3m} be a space of composition $(X_{2m} \times X_m)$ and let define the compositions $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ to obtain a thriad compositions.

Theorem 4. *If in Weyl space W_{3m} any of compositions in the thriad compositions $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ are of the type$(ch - ch)$, then the space W_{3m} is Riemannian.*

Proof. We will make our considerations in adapted with the composition $(X_{2m} \times X_m)$ coordinate system. According to (33) the equality (34) can be write in the form

$$\begin{array}{lll} \partial_i g_{pq} = 2\omega_i\ g_{pq} & \partial_p g_{ij} = 2\omega_p\ g_{ij} & \partial_a g_{ij} = 2\omega_a g_{ij} \\ \partial_i g_{ab} = 2\omega_i\ g_{ab} & \partial_p g_{ab} = 2\omega_p\ g_{ab} & \partial_a g_{pq} = 2\omega_a g_{pq} \\ \partial_i g_{pa} = 2\omega_i\ g_{pa} & \partial_p g_{ia} = 2\omega_p\ g_{ia} & \partial_a g_{ip} = 2\omega_a g_{ip}\ . \end{array} \tag{35}$$

From (35) it follows

$$\begin{array}{lll} g_{ij} = f g_{ij}(u^i) & g_{pq} = f g_{pq}(u^p) & g_{ab} = f g_{ab}(u^a) \\ g_{ia} = f g_{ia}(u^i, u^a) & g_{iq} = f g_{ip}(u^i, u^p) & g_{pa} = f g_{pa}(u^p, u^a), \end{array} \tag{36}$$

where f is an arbitrary function.

According to (35), (36) $\omega_i = \dfrac{1}{2}\ \partial_i \ln f$, $\omega_p = \dfrac{1}{2}\ \partial_p \ln f$, $\omega_a = \dfrac{1}{2}\ \partial_a \ln f$, i.e. $\omega_\alpha = \dfrac{1}{2}\ \partial_\alpha \ln f$. That means ω_α is a gradient vector. Therefore the space $W_{3m}(g_{\alpha\beta}, \omega_\sigma)$ is Riemannian.

3.2. Let the compositions $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ be $(g - g)$ compositions in the affinely connected space A_{3m} with a symmetric connection.

Introduce the following tensors

$$E^\nu_{\alpha\beta} = a^\sigma_\alpha A^\nu_{\beta\sigma} + a^\sigma_\beta A^\nu_{\sigma\alpha}, \ \ M^\nu_{\alpha\beta} = b^\sigma_\alpha B^\nu_{\beta\sigma} + b^\sigma_\beta B^\nu_{\sigma\alpha}, \ \ N^\nu_{\alpha\beta} = c^\sigma_\alpha C^\nu_{\beta\sigma} + c^\sigma_\beta C^\nu_{\sigma\alpha}. \tag{37}$$

Theorem 5. *Any composition of the thriad compositions* $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ *is* $(g-g)$ *composition in the affinely connected space* A_{3m} *if and only if*

$$E^{\nu}_{\alpha\beta} = M^{\nu}_{\alpha\beta} = N^{\nu}_{\alpha\beta} = 0 \ , \tag{38}$$

or in the adapted with the composition $(X_{2m} \times X_m)$ *coordinate system if and only if the coefficients of the connection satisfy*

$$\Gamma^l_{pq} = \Gamma^l_{pa} = \Gamma^l_{ab} = \Gamma^p_{ij} = \Gamma^p_{ia} = \Gamma^p_{ab} = \Gamma^c_{ij} = \Gamma^c_{pq} = \Gamma^c_{ip} = 0. \tag{39}$$

Proof. According to (13) the compositions of the thriad compositions are $(g-g)$ compositions if and only if the equalities (38) are hold. Let continue our considerations in the adapted with the composition $(X_{2m} \times X_m)$ coordinate system. Because of [3] $E^{\nu}_{\alpha\beta} = 0$ is equivalent to

$$\Gamma^l_{ab} = \Gamma^p_{ab} = \Gamma^c_{ij} = \Gamma^c_{ip} = \Gamma^c_{pq} = 0 \ . \tag{40}$$

From (28),(31),(37) we obtain

$$\begin{gathered} M^l_{ij} = M^l_{ip} = M^l_{pi} = M^l_{ai} = M^l_{ia} = 0, \\ M^p_{qi} = M^p_{iq} = M^p_{sq} = M^p_{al} = M^p_{la} = M^p_{ab} = M^p_{qa} = M^p_{aq} = 0, \\ M^c_{pi} = M^c_{ip} = M^c_{pq} = M^c_{ia} = M^c_{ai} = M^c_{ab} = M^c_{pa} = M^c_{ap} = 0, \\ M^l_{pq} = 4\Gamma^l_{qp},\ M^l_{ab} = 4\Gamma^l_{ba},\ M^l_{sa} = M^l_{as} = 4\Gamma^l_{sa},\ M^p_{ji} = M^c_{ij} = 4\Gamma^c_{ji} \ ; \end{gathered} \tag{41}$$

$$\begin{gathered} N^l_{ij} = N^l_{ip} = N^l_{pi} = N^l_{ai} = N^l_{ia} = N^l_{ab} = N^l_{pa} = N^l_{ap} = 0, \\ N^p_{qi} = N^p_{iq} = N^p_{sq} = N^p_{qa} = N^p_{as} = 0, \\ N^c_{ij} = N^c_{pl} = N^c_{lp} = N^c_{ia} = N^c_{ai} = N^c_{ab} = N^c_{pa} = N^c_{ap} = 0, \\ N^l_{pq}{=}4\Gamma^l_{qp},\ N^p_{ij}{=}4\Gamma^p_{ji},\ N^p_{al}{=}N^p_{la}{=}4\Gamma^p_{al},\ N^p_{ab}{=}4\Gamma^p_{ba},\ N^c_{pq}{=}4\Gamma^c_{qp} \ . \end{gathered} \tag{42}$$

According to (41) $M^{\nu}_{\alpha\beta} = 0$ is equivalent to

$$\Gamma^l_{pq} = \Gamma^l_{ab} = \Gamma^l_{pa} = \Gamma^p_{ij} = \Gamma^c_{ij} = 0 \tag{43}$$

and according to (42) $N^{\nu}_{\alpha\beta} = 0$ is equivalent to

$$\Gamma^l_{pq} = \Gamma^p_{ia} = \Gamma^p_{ab} = \Gamma^p_{ij} = \Gamma^c_{pq} = 0. \tag{44}$$

From (40), (43), (44) follows (39).

Corollary 3. *If any composition of the thriad compositions* $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ *is* $(g-g)$ *composition in the affinely connected space* A_{3m} *then in the adapted with the composition* $(X_{2m} \times X_m)$ *coordinate system the components of the tensor of the curvature* $R_{\alpha\beta\sigma}{}^{\nu}_{\cdot}$ *of the space* A_{3m} *satisfy*

$$R_{pqs}{}^{l}_{\cdot} = R_{abc}{}^{l}_{\cdot} = R_{pqa}{}^{l}_{\cdot} = R_{paq}{}^{l}_{\cdot} = R_{apq}{}^{l}_{\cdot} = R_{abp}{}^{l}_{\cdot} = R_{apb}{}^{l}_{\cdot} = R_{pab}{}^{l}_{\cdot} = 0,$$

$$R_{ijk}{}^{p}_{\cdot} = R_{abc}{}^{p}_{\cdot} = R_{ija}{}^{p}_{\cdot} = R_{iaj}{}^{p}_{\cdot} = R_{aij}{}^{p}_{\cdot} = R_{abi}{}^{p}_{\cdot} = R_{aib}{}^{p}_{\cdot} = R_{iab}{}^{p}_{\cdot} = 0,$$

$$R_{ijk}{}^{c}_{\cdot} = R_{pqb}{}^{c}_{\cdot} = R_{ijp}{}^{c}_{\cdot} = R_{ipj}{}^{c}_{\cdot} = R_{pij}{}^{c}_{\cdot} = R_{pqi}{}^{c}_{\cdot} = R_{piq}{}^{c}_{\cdot} = R_{ipq}{}^{c}_{\cdot} = 0.$$

The proof follows immediately from (39) and [1] ($R_{\alpha\beta\sigma}{}^{\nu}_{\cdot} = \partial_\alpha \Gamma^{\nu}_{\beta\sigma} - \partial_\beta \Gamma^{\nu}_{\alpha\sigma} - \Gamma^{\nu}_{\alpha\delta}\Gamma^{\delta}_{\beta\sigma} - \Gamma^{\nu}_{\beta\delta}\Gamma^{\delta}_{\alpha\sigma}$).

4. Bundles of manifolds in A_{3m}

Let the thriad compositions $(X_{2m} \times X_m)$, $(Y_{2m} \times Y_m)$, $(Z_{2m} \times Z_m)$ be given in A_{3m}. Consider the topological product $X_m \times Y_m \times Z_m$. Three positions $P(X_m), P(Y_m), P(Z_m)$ of the manifolds X_m, Y_m, Z_m will pass through any point of the space A_{3m}. We shall call the topological product $X_m \times Y_m \times Z_m$ a bundle of manifolds and we shall notice it S_3^{mmm}. The special bundles S_3^{mmm} will be noticed as well as the special compositions. For example S_3^{mmm} is of the type (ch, Y_m, Z_m) when the position $P(X_m)$ is parallelly translated along lines of Y_m and Z_m or S_3^{mmm} is of the type (X_m, g, Z_m) when the position $P(Y_m)$ is parallelly translated along lines of Y_m.

Consider the equalities

$$\overset{m}{a}{}^{\delta}_{\alpha}\, \overset{m}{a}{}^{\nu}_{\sigma}\, \overset{m}{A}{}^{\beta}_{\delta\nu} = 0, \quad \overset{m}{b}{}^{\delta}_{\alpha}\, \overset{m}{b}{}^{\nu}_{\sigma}\, \overset{m}{B}{}^{\beta}_{\delta\nu} = 0, \quad \overset{m}{c}{}^{\delta}_{\alpha}\, \overset{m}{c}{}^{\nu}_{\sigma}\, \overset{m}{C}{}^{\beta}_{\delta\nu} = 0; \tag{45}$$

$$\overset{n}{a}{}^{\delta}_{\alpha}\, \overset{m}{a}{}^{\nu}_{\sigma}\, \overset{m}{A}{}^{\beta}_{\delta\nu} = 0, \quad \overset{n}{b}{}^{\delta}_{\alpha}\, \overset{m}{b}{}^{\nu}_{\sigma}\, \overset{m}{B}{}^{\beta}_{\delta\nu} = 0, \quad \overset{n}{c}{}^{\delta}_{\alpha}\, \overset{m}{c}{}^{\nu}_{\sigma}\, \overset{m}{C}{}^{\beta}_{\delta\nu} = 0; \tag{46}$$

and

$$\overset{m}{a}{}^{\delta}_{\sigma}\, \overset{n}{A}{}^{\beta}_{\alpha\delta} = 0, \quad \overset{m}{b}{}^{\delta}_{\sigma}\, \overset{n}{B}{}^{\beta}_{\alpha\delta} = 0, \quad \overset{m}{c}{}^{\delta}_{\sigma}\, \overset{n}{C}{}^{\beta}_{\alpha\delta} = 0. \tag{47}$$

According to (3),(7),(29),(30) the equalities (45), (46) and (47) are equivalent to

$$\begin{aligned} &A^{\beta}_{\alpha\sigma} - a^{\nu}_{\sigma}\, A^{\beta}_{\alpha\nu} - a^{\nu}_{\alpha}\, A^{\beta}_{\nu\sigma} + a^{\rho}_{\alpha}\, a^{\nu}_{\sigma}\, A^{\beta}_{\rho\nu} = 0,\\ &B^{\beta}_{\alpha\sigma} - b^{\nu}_{\sigma}\, B^{\beta}_{\alpha\nu} - b^{\nu}_{\alpha}\, B^{\beta}_{\nu\sigma} + b^{\rho}_{\alpha}\, b^{\nu}_{\sigma}\, B^{\beta}_{\rho\nu} = 0,\\ &C^{\beta}_{\alpha\sigma} - c^{\nu}_{\sigma}\, C^{\beta}_{\alpha\nu} - c^{\nu}_{\alpha}\, C^{\beta}_{\nu\sigma} + c^{\rho}_{\alpha}\, c^{\nu}_{\sigma}\, C^{\beta}_{\rho\nu} = 0; \end{aligned} \tag{48}$$

$$
\begin{aligned}
&A^{\beta}_{\alpha\sigma} - a^{\nu}_{\sigma}\ A^{\beta}_{\alpha\nu} + a^{\nu}_{\alpha}\ A^{\beta}_{\nu\sigma} - a^{\rho}_{\alpha}\ a^{\nu}_{\sigma}\ A^{\beta}_{\rho\nu} = 0,\\
&B^{\beta}_{\alpha\sigma} - b^{\nu}_{\sigma}\ B^{\beta}_{\alpha\nu} + b^{\nu}_{\alpha}\ B^{\beta}_{\nu\sigma} - b^{\rho}_{\alpha}\ b^{\nu}_{\sigma}\ B^{\beta}_{\rho\nu} = 0,\\
&C^{\beta}_{\alpha\sigma} - c^{\nu}_{\sigma}\ C^{\beta}_{\alpha\nu} + c^{\nu}_{\alpha}\ C^{\beta}_{\nu\sigma} - c^{\rho}_{\alpha}\ c^{\nu}_{\sigma}\ C^{\beta}_{\rho\nu} = 0;
\end{aligned} \tag{49}
$$

and

$$
A^{\beta}_{\alpha\sigma} - a\ {}^{\delta}_{\sigma}\ A^{\beta}_{\alpha\delta} = 0,\quad B^{\beta}_{\alpha\sigma} - b\ {}^{\delta}_{\sigma}\ B^{\beta}_{\alpha\delta} = 0,\quad C^{\beta}_{\alpha\sigma} - c\ {}^{\delta}_{\sigma}\ C^{\beta}_{\alpha\delta} = 0, \tag{50}
$$

respectively.

Introduce the tensors

$$
\begin{aligned}
&P^{\beta}_{\alpha\sigma} = A^{\beta}_{\alpha\sigma} - a^{\nu}_{\sigma}\ A^{\beta}_{\alpha\nu} - a^{\nu}_{\alpha}\ A^{\beta}_{\nu\sigma} + a^{\rho}_{\alpha}\ a^{\nu}_{\sigma}\ A^{\beta}_{\rho\nu},\\
&Q^{\beta}_{\alpha\sigma} = B^{\beta}_{\alpha\sigma} - b^{\nu}_{\sigma}\ B^{\beta}_{\alpha\nu} - b^{\nu}_{\alpha}\ B^{\beta}_{\nu\sigma} + b^{\rho}_{\alpha}\ b^{\nu}_{\sigma}\ B^{\beta}_{\rho\nu},\\
&L^{\beta}_{\alpha\sigma} = C^{\beta}_{\alpha\sigma} - c^{\nu}_{\sigma}\ C^{\beta}_{\alpha\nu} - c^{\nu}_{\alpha}\ C^{\beta}_{\nu\sigma} + c^{\rho}_{\alpha}\ c^{\nu}_{\sigma}\ C^{\beta}_{\rho\nu};
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
&K^{\beta}_{\alpha\sigma} = A^{\beta}_{\alpha\sigma} - a^{\nu}_{\sigma}\ A^{\beta}_{\alpha\nu} + a^{\nu}_{\alpha}\ A^{\beta}_{\nu\sigma} - a^{\rho}_{\alpha}\ a^{\nu}_{\sigma}\ A^{\beta}_{\rho\nu},\\
&T^{\beta}_{\alpha\sigma} = B^{\beta}_{\alpha\sigma} - b^{\nu}_{\sigma}\ B^{\beta}_{\alpha\nu} + b^{\nu}_{\alpha}\ B^{\beta}_{\nu\sigma} - b^{\rho}_{\alpha}\ b^{\nu}_{\sigma}\ B^{\beta}_{\rho\nu},\\
&F^{\beta}_{\alpha\sigma} = C^{\beta}_{\alpha\sigma} - c^{\nu}_{\sigma}\ C^{\beta}_{\alpha\nu} + c^{\nu}_{\alpha}\ C^{\beta}_{\nu\sigma} - c^{\rho}_{\alpha}\ c^{\nu}_{\sigma}\ C^{\beta}_{\rho\nu};
\end{aligned} \tag{52}
$$

and

$$
U^{\beta}_{\alpha\sigma} = A^{\beta}_{\alpha\sigma} - a\ {}^{\delta}_{\sigma}\ A^{\beta}_{\alpha\delta},\quad V^{\beta}_{\alpha\sigma} = B^{\beta}_{\alpha\sigma} - b\ {}^{\delta}_{\sigma}\ B^{\beta}_{\alpha\delta},\quad W^{\beta}_{\alpha\sigma} = C^{\beta}_{\alpha\sigma} - c\ {}^{\delta}_{\sigma}\ C^{\beta}_{\alpha\delta}\ . \tag{53}
$$

Then (45), (46) and (47) are equivalent to

$$
P^{\beta}_{\alpha\sigma} = 0,\ Q^{\beta}_{\alpha\sigma} = 0,\ L^{\beta}_{\alpha\sigma} = 0; \tag{54}
$$

$$
K^{\beta}_{\alpha\sigma} = 0,\ T^{\beta}_{\alpha\sigma} = 0,\ F^{\beta}_{\alpha\sigma} = 0; \tag{55}
$$

and

$$
U^{\beta}_{\alpha\sigma} = 0,\ V^{\beta}_{\alpha\sigma} = 0,\ W^{\beta}_{\alpha\sigma} = 0, \tag{56}
$$

respectively.

Invariant characteristics for the special bundles S_3^{mmm} in A_{3m} are written in the following table

N	special bundle	invariant characteristic	invariant characteristic in adapted with composition $(X_{2m}\times X_m)$ coordinate system
1	(ch, Y_m, Z_m)	$K^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ia}=\Gamma^l_{pa}=\Gamma^p_{ia}=\Gamma^p_{qa}=0$
2	(X_m, ch, Z_m)	$T^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{qi}=\Gamma^p_{ai}=\Gamma^c_{pi}=\Gamma^c_{ai}=0$
3	(X_m, Y_m, ch)	$F^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ip}=\Gamma^l_{ap}=\Gamma^c_{ap}=\Gamma^c_{ip}=0$
4	(g, Y_m, Z_m)	$P^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ab}=\Gamma^p_{ab}=0$
5	(X_m, g, Z_m)	$Q^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{ij}=\Gamma^c_{ij}=0$
6	(X_m, Y_m, g)	$L^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{pq}=\Gamma^c_{pq}=0$
7	(d, Y_m, Z_m)	$U^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{a\sigma}=\Gamma^p_{a\sigma}=0$
8	(X_m, d, Z_m)	$V^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{i\sigma}=\Gamma^c_{i\sigma}=0$
9	(X_m, Y_m, d)	$W^{\beta}_{\alpha}=0$	$\Gamma^l_{p\sigma}=\Gamma^c_{p\sigma}=0$
10	(ch, g, Z_m)	$K^{\beta}_{\alpha\sigma}=0, Q^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ia}=\Gamma^l_{pa}=\Gamma^p_{ia}=\Gamma^p_{qa}=\Gamma^p_{ij}=\Gamma^c_{ij}=0$
11	(g, ch, Z_m)	$T^{\beta}_{\alpha\sigma}=0, P^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{qi}=\Gamma^p_{ai}=\Gamma^c_{pi}=\Gamma^c_{ai}=\Gamma^l_{ab}=\Gamma^p_{ab}=0$
12	(X_m, ch, g)	$T^{\beta}_{\alpha\sigma}=0, L^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{qi}=\Gamma^p_{ai}=\Gamma^c_{pi}=\Gamma^c_{ai}=\Gamma^l_{pq}=\Gamma^c_{pq}=0$
13	(X_m, g, ch)	$F^{\beta}_{\alpha\sigma}=0, Q^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ip}=\Gamma^l_{ap}=\Gamma^c_{ap}=\Gamma^c_{ip}=\Gamma^p_{ij}=\Gamma^c_{ij}=0$
14	(ch, Y_m, g)	$K^{\beta}_{\alpha\sigma}=0, L^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ia}=\Gamma^l_{ap}=\Gamma^p_{ia}=\Gamma^p_{qa}=\Gamma^l_{pq}=\Gamma^c_{pq}=0$
15	(g, Y_m, ch)	$F^{\beta}_{\alpha\sigma}=0, P^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ip}=\Gamma^l_{ap}=\Gamma^c_{ap}=\Gamma^c_{ip}=\Gamma^l_{ab}=\Gamma^p_{ab}=0$
16	(ch, d, Z_m)	$K^{\beta}_{\alpha\sigma}=0, V^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ia}=\Gamma^l_{pa}=\Gamma^p_{ia}=\Gamma^p_{qa}=\Gamma^p_{i\alpha}=\Gamma^c_{i\alpha}=0$
17	(d, ch, Z_m)	$T^{\beta}_{\alpha\sigma}=0, U^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{qi}=\Gamma^p_{ai}=\Gamma^c_{pi}=\Gamma^c_{ai}=\Gamma^l_{a\beta}=\Gamma^p_{a\beta}=0$
18	(ch, Y_m, d)	$K^{\beta}_{\alpha\sigma}=0, W^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ia}=\Gamma^l_{ap}=\Gamma^p_{ia}=\Gamma^p_{qa}=\Gamma^l_{p\alpha}=\Gamma^c_{p\alpha}=0$
19	(d, Y_m, ch)	$U^{\beta}_{\alpha\sigma}=0, F^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ip}=\Gamma^l_{ap}=\Gamma^c_{ap}=\Gamma^c_{ip}=\Gamma^l_{a\beta}=\Gamma^p_{a\beta}=0$
20	(X_m, ch, d)	$T^{\beta}_{\alpha\sigma}=0, W^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{qi}=\Gamma^p_{ai}=\Gamma^c_{pi}=\Gamma^c_{ai}=\Gamma^l_{p\alpha}=\Gamma^c_{p\alpha}=0$
21	(X_m, d, ch)	$F^{\beta}_{\alpha\sigma}=0, V^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ip}=\Gamma^l_{ap}=\Gamma^c_{ap}=\Gamma^c_{ip}=\Gamma^p_{i\alpha}=\Gamma^c_{i\alpha}=0$
22	(g, d, Z_m)	$P^{\beta}_{\alpha\sigma}=0, V^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ab}=\Gamma^p_{ab}=\Gamma^p_{i\alpha}=\Gamma^c_{i\alpha}=0$
23	(d, g, Z_m)	$Q^{\beta}_{\alpha\sigma}=0, U^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{ij}=\Gamma^c_{ij}=\Gamma^l_{a\beta}=\Gamma^p_{a\beta}=0$
24	(g, Y_m, d)	$P^{\beta}_{\alpha\sigma}=0, W^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ab}=\Gamma^p_{ab}=\Gamma^l_{p\alpha}=\Gamma^c_{p\alpha}=0$
25	(d, Y_m, g)	$U^{\beta}_{\alpha\sigma}=0, L^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{pq}=\Gamma^c_{pq}=\Gamma^l_{a\beta}=\Gamma^p_{a\beta}=0$
26	(X_m, g, d)	$Q^{\beta}_{\alpha\sigma}=0, W^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{ij}=\Gamma^c_{ij}=\Gamma^l_{p\alpha}=\Gamma^c_{p\alpha}=0$
27	(X_m, d, g)	$L^{\beta}_{\alpha\sigma}=0, V^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{pq}=\Gamma^c_{pq}=\Gamma^p_{i\alpha}=\Gamma^c_{i\alpha}=0$
28	(g, g, d)	$P^{\beta}_{\alpha\sigma}=0, Q^{\beta}_{\alpha\sigma}=0, W^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ab}=\Gamma^p_{ab}=\Gamma^p_{ij}=\Gamma^c_{ij}=\Gamma^l_{p\alpha}=\Gamma^c_{p\alpha}=0$
29	(g, d, g)	$P^{\beta}_{\alpha\sigma}=0, L^{\beta}_{\alpha\sigma}=0, V^{\beta}_{\alpha\sigma}=0$	$\Gamma^l_{ab}=\Gamma^p_{ab}=\Gamma^l_{pq}=\Gamma^c_{pq}=\Gamma^p_{i\alpha}=\Gamma^c_{i\alpha}=0$
30	(d, g, g)	$Q^{\beta}_{\alpha\sigma}=0, L^{\beta}_{\alpha\sigma}=0, U^{\beta}_{\alpha\sigma}=0$	$\Gamma^p_{ij}=\Gamma^c_{ij}=\Gamma^l_{pq}=\Gamma^c_{pq}=\Gamma^i_{a\beta}=\Gamma^p_{i\beta}=0$

N	special bundle	invariant characteristic	invariant characteristic in adapted with composition $(X_{2m}\times X_m)$ coordinate system
31	(ch,g,d)	$K^{\beta}_{\alpha\sigma}=0, Q^{\beta}_{\alpha\sigma}=0, W^{\beta}_{\alpha\sigma}=0$	$\Gamma^{l}_{ia}=\Gamma^{l}_{pa}=\Gamma^{p}_{ia}=\Gamma^{p}_{qa}=\Gamma^{p}_{ij}=\Gamma^{c}_{ij}=\Gamma^{l}_{p\alpha}=\Gamma^{p}_{a\beta}=0$
32	(g,ch,d)	$T^{\beta}_{\alpha\sigma}=0, P^{\beta}_{\alpha\sigma}=0, W^{\beta}_{\alpha\sigma}=0$	$\Gamma^{P}_{qi}=\Gamma^{p}_{ai}=\Gamma^{c}_{pi}=\Gamma^{c}_{ai}=\Gamma^{l}_{ab}=\Gamma^{p}_{ab}=\Gamma^{l}_{p\alpha}=\Gamma^{p}_{a\beta}=0$
33	(ch,d,g)	$K^{\beta}_{\alpha\sigma}=0, L^{\beta}_{\alpha\sigma}=0, V^{\beta}_{\alpha\sigma}=0$	$\Gamma^{l}_{ia}=\Gamma^{l}_{pa}=\Gamma^{p}_{ia}=\Gamma^{p}_{qa}=\Gamma^{l}_{pq}=\Gamma^{c}_{pq}=\Gamma^{p}_{i\alpha}=\Gamma^{c}_{i\beta}=0$
34	(g,d,ch)	$P^{\beta}_{\alpha\sigma}=0, F^{\beta}_{\alpha\sigma}=0, V^{\beta}_{\alpha\sigma}=0$	$\Gamma^{l}_{ip}=\Gamma^{l}_{ap}=\Gamma^{c}_{ap}=\Gamma^{c}_{ip}=\Gamma^{l}_{ab}=\Gamma^{p}_{ab}=\Gamma^{p}_{i\alpha}=\Gamma^{c}_{i\beta}=0$
35	(d,ch,g)	$T^{\beta}_{\alpha\sigma}=0, L^{\beta}_{\alpha\sigma}=0, U^{\beta}_{\alpha\sigma}=0$	$\Gamma^{p}_{qi}=\Gamma^{p}_{ai}=\Gamma^{c}_{pi}=\Gamma^{c}_{ai}=\Gamma^{l}_{pq}=\Gamma^{c}_{pq}=\Gamma^{l}_{a\beta}=\Gamma^{p}_{a\beta}=0$
36	(d,g,ch)	$F^{\beta}_{\alpha\sigma}=0, Q^{\beta}_{\alpha\sigma}=0, U^{\beta}_{\alpha\sigma}=0$	$\Gamma^{l}_{ip}=\Gamma^{l}_{pa}=\Gamma^{c}_{pa}=\Gamma^{c}_{ip}=\Gamma^{p}_{ij}=\Gamma^{c}_{ij}=\Gamma^{l}_{a\beta}=\Gamma^{p}_{a\beta}=0$

We shall prove the proposition N32 from the table. Let S_3^{mmm} is a bundle of the type (g,ch,d) i.e. the position $P(X_m)$ is parallelly translated along the lines of X_m, the position $P(Y_m)$ is parallelly translated along the lines of X_m and Z_m, the position $P(Z_m)$ is parallelly translated along all lines of A_{3m}. It means that the compositions $(X_{2m}\times X_m)$, $(Y_{2m}\times Y_m)$, $(Z_{2m}\times Z_m)$ are of type (X_{2m},g), (Y_{2m},ch), (Z_{2m},d), respectively. According to [3] these three compositions are of the indicated type if and only if

$$\overset{m}{a}{}^{\delta}_{\alpha}\,\overset{m}{a}{}^{\nu}_{\sigma}\,\overset{m}{A}{}^{\beta}_{\delta\nu}=0,\quad \overset{n}{b}{}^{\delta}_{\alpha}\,\overset{m}{b}{}^{\nu}_{\sigma}\,\overset{m}{B}{}^{\beta}_{\delta\nu}=0,\quad \overset{m}{c}{}^{\sigma}_{\nu}\,\overset{n}{C}{}^{\beta}_{\alpha\sigma}=0\ . \tag{57}$$

From (45),(46),(47),(48),(49),(50), (51),(52), (53), (54), (55), (56) it follows that (57) are equivalent to $T^{\beta}_{\alpha\sigma}=0,\ P^{\beta}_{\alpha\sigma}=0,\ W^{\beta}_{\alpha\sigma}=0$.

Let consider the adapted with the composition $(X_{2m}\times X_m)$ coordinate system. From (7), (24), (26), (28), (29), (30), (31), (40), (49), (50),(51),(52),(56) we obtain $\Gamma^{P}_{qi}=\Gamma^{p}_{ai}=\Gamma^{c}_{pi}=\Gamma^{c}_{ai}=\Gamma^{l}_{ab}=\Gamma^{p}_{ab}=\Gamma^{l}_{p\alpha}=\Gamma^{p}_{a\beta}=0$.

Let consider the case when $m=3$ and let the space A_3 is the space of a composition $(X_2\times X_1)$. Define also the compositions $(Y_2\times Y_1)$, $(Z_2\times Z_1)$ and the bundle $S_3^{111}(X_1\times Y_1\times Z_1)$. If $\underset{1}{v}^i$, $\underset{2}{v}^i$, $\underset{3}{v}^i$ are eigen-vectors of the matrix (a^{β}_{α}), then $\underset{1}{v}^i\in P(X_1)$, $\underset{2}{v}^i\in P(Y_1)$, $\underset{3}{v}^i\in P(Z_1)$. The vectors $\underset{1}{v}^i$, $\underset{2}{v}^i$, $\underset{3}{v}^i$ define a net $(\underset{1}{v}^i,\underset{2}{v}^i,\underset{3}{v}^i)\in A_3$. Now the bundle S_3^{111} is of the type (ch,ch,ch) or (g,g,g) if and only if the net $(\underset{1}{v}^i,\underset{2}{v}^i,\underset{3}{v}^i)$ is Chebishevian or geodesic, respectively.

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ТРОЙКА КОМПОЗИЦИИ В ПРОСТРАНСТВО С АФИННА СВЪРЗАНОСТ A_{3m}

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Резюме. Нека пространството с афинна свързаност без торзия A_{3m} е пространство от композиция $(X_{2m} \times X_m)$, породена от афинора a_α^β. Въвеждаме два нови афинора b_α^β и c_α^β, свързани с афинора a_α^β. Разгледани са две нови композиции $(Y_{2m} \times Y_m)$ и $(Z_{2m} \times Z_m)$, породени съответно от афинорите b_α^β и c_α^β, и така е определена тройка композиции.

Доказано е, че ако две композиции от тройката композиции са специални от типа $(d - d)$ или $(ch - ch)$, то и третата композиция е от същия тип. Намерени са характеристики на специалните тройки композиции. Определен е видът на пространствата A_{3m}, които съдържат специални тройки композиции, в адаптираната с композицията $(X_{2m} \times X_m)$ координатна система. Доказано е, че когато във вайловото пространство W_{3m} има тройка композиции от типа $(ch - ch)$, тогава то е риманово.

Топологичното произведение на многообразията $X_m \times Y_m \times Z_m$ е наречено сноп и е означено S_3^{mmm}. Намерени са характеристики на специалните снопове $S_3^{mmm} \in A_{3m}$ и е определен видът на пространствата A_{3m}, съдържащи такива специални снопове.